IDENTIFICATION OF RADAR SIGNALS USING DISCRIMINANT VECTORS[1]

Jan Matuszewski[2]

Abstract

One of the most principal functions of the electronic intelligence (ELINT) system is gathering the basic information from the entire electromagnetic spectrum and its analysis. The paper presents some aspects of radar signals acquisition in ELINT system, the analysis of their parameters, feature extraction using discriminant vectors and linear Karhunen-Loeve transformation and their applying to identification of the intercepted signals.

1. Introduction

In a radar signal recognition a great deal of information collected by the receivers is presented in real time. Specialized computer system must be used to the analysis, future extraction of the intercepted unknown signals. Conventional ELINT systems measure the basic parameters of intercepted radar signals. These basic (typical) parameters are as follows: radio frequency (RF), time of arrival (ToA), pulse width (PW), angle of arrival (AoA), amplitude (A) or pulse repetition interval (PRI). The radar signal classifier compares the measured signal's characteristics against a library of stored radar types, which may have a high degree of inherent uncertainty arising from the methods of data gathering and processing. Figure 1 illustrates the basic structure of classification (not identification) radar emission sources.

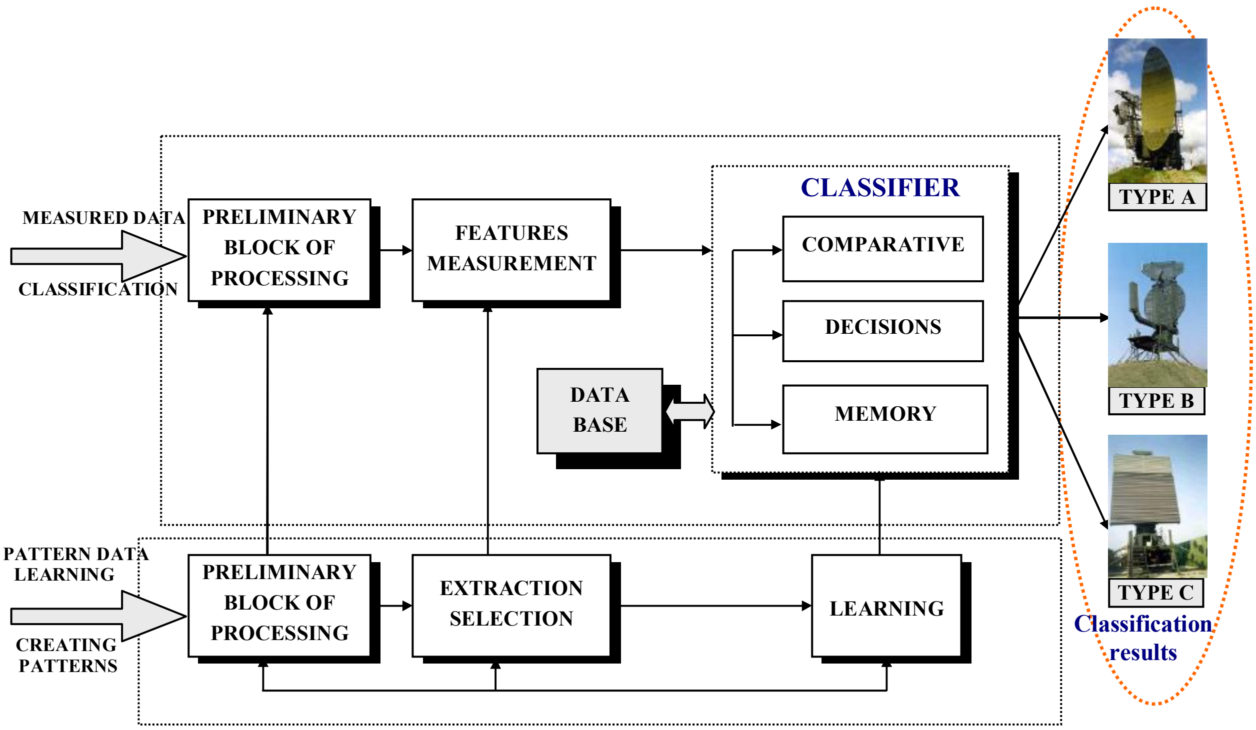

Fig. 1. The classic structure of recognition of radar type

[1] This paper is financed from sources for science in the years 2007-2010 in a frame of ordered scientific project PBZ-MNiSW-DBO-04/I/2007.

[2] Military University of Technology, Faculty of Electronics,
Institute of Radioelectronics, 2 S. Kaliskiego Str., 00-908, Poland
Phone: (0 22) 683 92 65
e-mail: jmatuszewski@wel.wat.edu.pl

The appropriate emitter database (DB) is one of the most important elements in the present electronic support measures (ESM) system. Specially important is utilization of an emitter DB in the process of identifying a detectable radar emissions. The characteristic of present battlefield electromagnetic environment, the process of acquisition and transformation data shows, that measured basic parameters are not enough during the process of source identification, (ref. [1, 5, 7]).

2. Discriminant vectors

Since each parameter can be ranked by its corresponding discriminatory potential that a subset of best features provides the optimal fitted subspace. The discriminant vectors can be used in such a classifier, in which patterns are projected from high dimensional data onto one-, two- or three dimensional space, so we can both observe the inherent structure of the data and design piecewise linear decision rule for signal classification, (ref. [2, 6]).

According to available bibliography concerning the subject matter, the emitter pattern ought to possess measured and non-measured features (information) such as.: logical, numerical, descriptive and graphical. The measured feature is a result of measurements and calculations, (ref. [1, 5, 7]). The non-measured feature can be expressed by the chain of words or logical expression. All information described an emission source can be divided as fallows:

- an *information in continuous manner variable* can accept facultative values from selected section of real numbers' axis;
- a *discreet information group of first kind* accepts discreet values such as 0/1 (yes/not). These features are expressed by qualitative character of occurrence or lack of certain properties;
- a *discreet information of second kind* is characterized by not-derivative elements of an object's structure, expressed by words or characters chain.

In a further part of this paper the radar signal consisting of N parameters will be assumed as a point in the N-dimensional Euclidean feature space denoted as a vector $\boldsymbol{x}$ or in the case of many samples by $\boldsymbol{x}_i$, where index i denotes the number of the sample. These vectors create a pattern of radar emission source which is called a "radar signature" in a database. In this way, the emitter database should include the patterns of radio-electronic devices and the technical characteristics of electromagnetic sources.

In pattern recognition the discriminant Fisher criterion is often used as the optimal linear method to transform the N-dimensional sets data from L classes onto optimal direction $\boldsymbol{d}$, (ref. [2, 4, 6]). We are looking for such a direction $\boldsymbol{d}$ in N-space defined by a set of measurements, on which orthogonally projected samples from L classes are maximally discriminated.

Assuming some simplification, the extended Fisher criterion (ref. [2, 6]) is related to the ratio of the projected class differences to the weighted sum of a within-class scatter (covariance) along axis $\boldsymbol{d}$, i.e.,

$$F_{\theta}(\boldsymbol{d}) = \frac{\boldsymbol{d}^T \boldsymbol{B}_{\theta}\, \boldsymbol{d}}{\boldsymbol{d}^T \boldsymbol{\Sigma}\, \boldsymbol{d}} \tag{1}$$

where:

$$\boldsymbol{B}_{\theta} = \sum_{i=1}^{L} \boldsymbol{B}_{i\theta} = \sum_{i=1}^{L} \boldsymbol{\Delta}_{i\theta}\, \boldsymbol{\Delta}_{i\theta}^{T} \tag{2}$$

$$\boldsymbol{\Delta}_{i\theta} = \boldsymbol{\mu}_i - \boldsymbol{\theta}; \qquad \boldsymbol{\theta} = \boldsymbol{\mu} = \sum_{i=1}^{L} \boldsymbol{\mu}_i \tag{3}$$

$$\boldsymbol{\Sigma}_i = \frac{1}{K_i} \sum_{k=1}^{K_i} (\boldsymbol{x}_{ik} - \boldsymbol{\mu}_i)\, (\boldsymbol{x}_{ik} - \boldsymbol{\mu}_i)^{T} \tag{4}$$

$\boldsymbol{\Sigma} = \sum_{i=1}^{L} \boldsymbol{\Sigma}_i$ - the sum of covariance matrix,

$\boldsymbol{B}_{\theta}$ - sum of the inter-class scatter (covariancematrix) for L classes,

$\boldsymbol{\Sigma}_i$ - within-class scatter (covariance matrix) for class i calculated in reference to vector $\boldsymbol{\theta}$,

$\boldsymbol{\theta}$ - the mean vector for L classes or constant value, (e.g. $\boldsymbol{\theta}=\boldsymbol{0}$),

$\boldsymbol{\Delta}_{i\theta}$ - the mean vector differences between class i and vector $\boldsymbol{\theta}$,

μ_i - the mean vector for class i;

K_i - number of samples $\boldsymbol{x}_k$ in class i.

The best direction $\boldsymbol{d}_1$, which maximizes (1), is obtained from solving the equation:

$$\boldsymbol{\Sigma}^{-1}\, \boldsymbol{B}\; \boldsymbol{d}_i = \lambda_i\, \boldsymbol{d}_i \tag{5}$$

where:

λ_i - the largest eigenvalue for the i feature,

$\boldsymbol{d}_i$ - the eigenvalue vector (discriminant vector) suitable to the largest eigenvalue for the i feature.

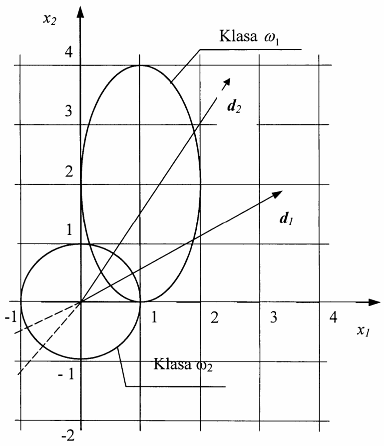

Fig. 2. Discriminants vectors d_1, d_2 for two classes ω_1, ω_2

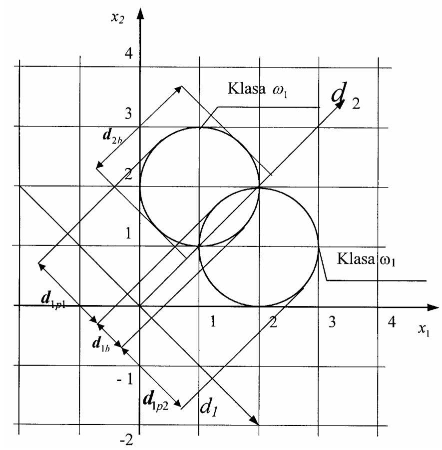

Fig. 3. Illustration of the false probability d_{1b} and d_{2b} according to direction d_1 and d_2

The next best direction $\boldsymbol{d}_2$ suit the successive largest eigenvalue calculated from the equation (5). For example, in order to classify the sample vector $\boldsymbol{x}_k$ representing radar signal to one with two classes a threshold Θ_i, must be specified and the following decision is made: decide class "1" if $\boldsymbol{d}\ \boldsymbol{x}_k \leq 0$, otherwise decide class "2". The threshold Θ_i can be calculated by applying the minimum risk criterion or any other criterion, which minimizes probability of error classification. In each of these events, the computation of the direction $\boldsymbol{d}_i$ and threshold Θ_i define a hyperplane which divides the N-space into two decision regions.

The probability of the false classification for example illustrated on Fig. 2 is equal 0,12 on the direction $\boldsymbol{d}_1$ and 0,16 on direction $\boldsymbol{d}_2$. For comparison for parameters x_1 and x_2 these probabilities are respectively equal 0,333 and 0,2. On Fig. 3 this probability are equal 0,2 on direction $\boldsymbol{d}_1$ and 1 on direction $\boldsymbol{d}_2$. For comparison for parameters x_1 and x_2 these probabilities are equal 0,333.

For multiclass problem (L>2) in n-space ($n \leq N$) we must compute $L(L-1)/2$ pairwise Fisher discriminants d_{ij} and their respective thresholds Θ_{ij}. The classifier is accomplished by orthogonally projecting the data onto the optimal Fisher discriminants and then designing the decision boundaries using piecewise-linear functions .

3. Linear Karhunen-Loeve transformation

Most of the feature extraction methods presented in the literature is based on the linear *Karhunen-Loeve transformation* or *eigenvector orthonormal expansion*, (ref. [3, 4, 6]). It is an optimal method in the mean square sense and it has a wide range of applications in reducing multi-dimensional data vectors by using only the first coefficients of this transformation.

In this procedure, the N-dimensional vectors $\boldsymbol{x}_k$ are transformed by multiplying by the eigenvectors $\boldsymbol{v}_j$ of estimated covariance matrix (within-class scatter) $\boldsymbol{A}$. The eigenvalues satisfy the equation

$$\boldsymbol{A}\ \boldsymbol{v}_j = \lambda_j\ \boldsymbol{v}_j\ ,\ j=\overline{1,N} \qquad (6)$$

where λ_j is the eigenvalue corresponding to the jth eigenvector.

For real data measured parameters of radar signals, $\boldsymbol{A}$ is a real symmetric matrix and all the eigenvectors are orthogonal, i.e., $\boldsymbol{v}_i^T\ \boldsymbol{v}_j = 0$ for $i \neq j$ and the eigenvalues are all greater than or equal to zero. Hence the eigenvalues can be ordered such that

$$\lambda_1 \leq \lambda_2 \leq, \ldots, \leq \lambda_N \qquad (7)$$

The matrix $\boldsymbol{V}=[\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_N]^T$ is an orthogonal transformation such that $\boldsymbol{y}_k = \boldsymbol{V}\boldsymbol{x}_x$ and each $y_j = \boldsymbol{v}_j^T\ \boldsymbol{x}_k$ is jth feature in a new space for the input vector $\boldsymbol{x}_k$. The mean square error obtained by projecting all the data into a subspace spanned by only a subset of K ($K \leq N$) eigenvectors can be minimized by choosing the eigenvectors corresponding to the K smallest eigenvalues λ_i. One of the advantages of calculating the eigenvalues is the possibility of using them to determine the "weighted coefficient" w_i for features.

These coefficients may be determined from the following condition:

If
$$\sum_{i=1}^{N} w_i = 1 \qquad (8)$$

then $$w_j = \frac{1}{\lambda_j}(\sum_{i=1}^{N} 1/\lambda_i)^{-1}, \quad j = \overline{1,N} \qquad (9)$$

The minimum long distance classifier calculates the weighted distance to each of L known classes and classifies the samples $\boldsymbol{x}_i$ to this j class, for which the following condition is fulfilled:

$$d(\boldsymbol{x}_i, \overline{\boldsymbol{x}}_l) = \min_j (\boldsymbol{x}_i - \overline{\boldsymbol{x}}_j)^{\boldsymbol{T}} \boldsymbol{V}^{\boldsymbol{T}} \boldsymbol{W}^{\boldsymbol{T}} \boldsymbol{W} \boldsymbol{V} (\boldsymbol{x}_i - \overline{\boldsymbol{x}}_j) \qquad (10)$$

where:

$\overline{\boldsymbol{x}}_j, \overline{\boldsymbol{x}}_l$ - the mean vectors for class j and l

$\boldsymbol{V}$ - the transform matrix consisting of eigenvectors $\boldsymbol{v}_i$,

$\boldsymbol{W}$ - the diagonal matrix of weighted coefficients.

4. Experiment

The minimum long distance classifier calculates the weighted distance to each of L known classes and classifies the samples $\boldsymbol{x}_i$ to this j class, for which the following condition is fulfilled:

$$d(\boldsymbol{x}_i, \overline{\boldsymbol{x}}_l) = \min_j (\boldsymbol{x}_i - \overline{\boldsymbol{x}}_j)^{\boldsymbol{T}} \boldsymbol{V}^{\boldsymbol{T}} \boldsymbol{W}^{\boldsymbol{T}} \boldsymbol{W} \boldsymbol{V} (\boldsymbol{x}_i - \overline{\boldsymbol{x}}_j) \qquad (11)$$

where:

$\overline{\boldsymbol{x}}_j, \overline{\boldsymbol{x}}_l$ - the mean vectors for class j and l;

$\boldsymbol{V}$ - the transform matrix consisting of disccriminant vectors $\boldsymbol{d}_i$ or eigenvectors $\boldsymbol{v}_i$;

$\boldsymbol{W}$ - the diagonal matrix of weight coefficients.

Calculations, which have been done on the sample consisting of radar signals from 125 classes, 4 features and 1500 data vectors, showed that using these weighted coefficients in minimum long distance classifier decreased the error probability of classification by about 10÷15 percent.

This classification of radar information, makes possible to accurately project radar signature during the process of its identification. This approach offers the possibility of creating the ELINT systems in optimal way without redundancy features. The process of identification an emission source depends on distinguishing the copy-type of a radar. It in the face of above mentioned, utilization of some specific properties of electronic devices functioning, e.g. radiated emission, can cause heightening probability of a correct identification.

5. Conclusions

In electromagnetic environment the values of radar signal parameters often change. Every time in a such case we have to calculate the new discriminant vectors, eigenvectors, eigenvalues and appropriate thresholds. The capability of ELINT/ESM system to correctly identify detectable radar emissions in the dense environment is a key to their application in the modern command, communication and control systems. The recording data and results of their analysis help us to extract some facts to constructing the knowledge base and design the expert systems in the last stage of radar identification.

References

[1] J. Dudczyk, J. Matuszewski, M. Wnuk, *The relational modelling approach to designing the emitter database.* Proceedings of the 3rd International Conference and Exposition. Complementarity of Ladar and Radar, Prague, Czech Republic, 22-23 April 2002.

[2] Luc Devroye, László Györfi, Gábor Lugosi, *A Probabilistic Theory of Pattern Recognition.* Springer_Verlag New York , Inc., 1996.

[3] W. Kwiatkowski, *Metody automatycznego rozpoznawania wzorców.* BEL Studio Sp. Z o.o. Warszawa 2001.

[4] J. Matuszewski, *Appplication of Eigenvalue Analysis and Discriminant Vectors in Radar Signal Recognition*, Proceedings of Nove smery v spracovani signalov III, Liptovsky Mikulas, Slovakia, 29-31 May 1996.

[5] J. Matuszewski, L. Paradowski, *The Knowledge Based Approach for Emitter Identification*, Conference Proceedings of 12th International Conference on Microwaves & Radar, MIKON'98, Kraków, Poland, 20-22 May 1998, vol. 3, pp. 810-814.

[6] W. Sobczak, W. Malina, *Metody selekcji i redukcji informacji*, WNT, Warszawa, 1985.

[7] R. G. Wiley, *Electronic Intelligence: The Analysis of Radar Signals*, Second Edition, Artech House, Boston, 1993, London, 1993.