

# Seismic Response of Vertically Irregular Frames: Response History and Modal Pushover Analyses

Chatpan Chintanapakdee<sup>1</sup> and Anil K. Chopra, M.ASCE<sup>2</sup>

**Abstract:** This study compares the seismic demands for vertically irregular and “regular” frames determined by rigorous nonlinear response history analysis (RHA), due to an ensemble of 20 ground motions. Forty-eight irregular frames, all 12-story high with strong columns and weak beams, were designed with three types of irregularities—stiffness, strength, and combined stiffness and strength—introduced in eight different locations along the height using two modification factors. The effects of vertical irregularity on the median values of story drifts and floor displacements are documented. Next, the median and dispersion values of the ratio of story drift demands determined by modal pushover analysis (MPA) and nonlinear RHA were computed to measure the bias and dispersion of MPA estimates leading to the following results: (1) the bias in the MPA procedure does not increase, i.e., its accuracy does not deteriorate, in spite of irregularity in stiffness, strength, or stiffness and strength provided the irregularity is in the middle or upper story, (2) the MPA procedure is less accurate relative to the regular frame in estimating the seismic demands of frames with strong or stiff-and-strong first story; soft, weak, or soft-and-weak lower half; stiff, strong, or stiff-and-strong lower half, (3) in spite of the larger bias in estimating drift demands for some of the stories in particular cases, the MPA procedure identifies stories with largest drift demands and estimates them to a sufficient degree of accuracy, detecting critical stories in such frames, and (4) the bias in the MPA procedure for frames with a soft, weak, or soft-and-weak first story is about the same as for the regular frame.

**DOI:** 10.1061/(ASCE)0733-9445(2004)130:8(1177)

**CE Database subject headings:** Building frames; Dynamic analysis; Ground motion; Inelastic action; Seismic response.

## Introduction

The seismic response of vertically irregular frames, the subject of numerous research investigations, was reviewed in two recent comprehensive investigations by Valmundsson and Nau (1997) and Al-Ali and Krawinkler (1998), both studies considering mass, stiffness, and strength irregularities separately and in various combinations. The first of these investigations focused on evaluating building code requirements for vertically irregular frame buildings, whereas the latter emphasized the effects of vertical irregularities on height-wise variation of seismic demands and behavior of frame buildings. It was found that among the four types of irregularity, the effect of mass irregularity is the smallest, the effect of strength irregularity is larger than the effect of stiffness irregularity, and the effect of combined-stiffness-and-strength irregularity is the largest. The roof displacement was shown to be a stable parameter not affected significantly by vertical irregularities (Al-Ali and Krawinkler 1998).

Both of these comprehensive investigations were based on idealized frames designed according to the strong-beam–weak-

column philosophy. Such a column-hinge model is likely to exaggerate the effects of irregularity by restricting the redistribution of yielding to the story that yields first and forms a story mechanism. Therefore, this paper studies the effects of vertical irregularities on seismic demands of frame buildings using the more realistic strong-column–weak-beam frame or beam-hinge model (Fig. 1); mass irregularity is not considered here because its effects are known to be small (Al-Ali and Krawinkler 1998).

Current structural engineering practice evaluates the seismic resistance of buildings using the nonlinear static procedure or pushover analysis described in FEMA-273 (BSSC 1997). Recently, modal pushover analysis (MPA) has been developed to improve conventional pushover procedures by including higher-mode contributions to seismic demands (Chopra and Goel 2002). This MPA procedure offers several attractive features. It retains the conceptual simplicity and computational attractiveness of current pushover procedures with invariant force distributions—now common in structural engineering practice. The computational effort involved in MPA including the first few—two or three—modes is comparable to that required in FEMA procedures using two or three lateral-force distributions. With the roof displacement determined from the elastic design spectrum and empirical equations for the ratio of peak deformations of inelastic and elastic systems, pushover analysis for each mode requires computational effort similar to one FEMA force distribution.

Although rooted in structural dynamics theory (Chopra and Goel 2002), MPA is based on three principal assumptions: (1) Coupling among modal coordinates arising from the yielding of the system can be neglected, (2) the peak response of the inelastic multi-degree-of-freedom (MDF) system associated with each modal force distribution can be determined by pushover analysis, and (3) the total response can be determined by combining the

<sup>1</sup>Lecturer, Dept. of Civil Engineering, Chulalongkorn Univ., Bangkok, Thailand.

<sup>2</sup>Johnson Professor, Dept. of Civil and Environmental Engineering, Univ. of California, Berkeley, California. E-mail: chopra@ce.berkeley.edu

Note. Associate Editor: Gregory A. MacRae. Discussion open until January 1, 2005. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on November 27, 2002; approved on September 16, 2003. This paper is part of the *Journal of Structural Engineering*, Vol. 130, No. 8, August 1, 2004. ©ASCE, ISSN 0733-9445/2004/8-1177-1185/\$18.00.



Fig. 1. Beam-hinge model of a 12-story frame

peak modal responses by standard modal combination rules. Each of these assumptions were justified in the original development of the theory underlying MPA. This procedure, although somewhat intuitive for inelastic buildings, is reduced to the standard response spectrum analysis procedure (Chopra 2001: Section 13.7) for elastic systems.

The accuracy of MPA must be evaluated for a wide range of structural systems and ground motions to identify the conditions under which it is applicable for seismic evaluation of structures. To this end, it has been applied to code-designed buildings (Goel and Chopra 2004), and generic frames (Chintanapakdee and Chopra 2003) designed according to the static force distribution specified in the International Building Code (IBC) (International Code Council 2000). By studying the bias and dispersion of this approximate procedure, MPA has been shown to be accurate enough in estimating seismic demands for the seismic evaluation of such “regular” buildings. As mentioned earlier, because vertical irregularities significantly influence the seismic demands on buildings, the next logical step is to determine whether MPA can estimate seismic demands on irregular buildings to a degree of accuracy sufficient for practical application. Furthermore, as all pushover analyses aim to detect any deficiency in the structure that results in localizing large seismic demands, MPA’s potential in this regard remains to be evaluated.

The objectives of this investigation are as follows: (1) To study the influence of vertical irregularities in the stiffness and strength distribution, separately and in combination, on seismic demands of strong-column–weak-beam frames by comparing the median seismic demands on irregular and regular frames computed by nonlinear response history analysis (RHA) for an ensemble of ground motions, and (2) to evaluate the accuracy of MPA in estimating seismic demands and detecting weakness in vertically irregular frames by documenting the bias and dispersion of the ratio of the seismic demands on irregular frames determined by MPA procedure to their “exact” values computed by nonlinear RHA.

The results presented in this paper show that the effects of vertical irregularities on seismic demands on a beam-hinge model (Fig. 1) of frames are significantly different than those reported using the less realistic column-hinge model (Al-Ali and Krawinkler 1998). It is also demonstrated that the MPA procedure has a similar degree of accuracy for estimating seismic demands for some types of irregular frames as it does for regular frames. In addition, the MPA procedure detects which stories will be subjected to large seismic demands; irregular frames for which MPA does not work well are identified.

## Structural Systems

### Reference Regular Frame

The MPA procedure has already been evaluated for regular frames of six different heights of 3, 6, 9, 12, 15, and 18 stories, each designed for five different strength levels. The second phase of the overall investigation concerned irregular frames, which is the subject of this paper. To focus on the issue of height-wise irregularity, the frame height was fixed at 12 stories, a midrise frame for which pushover analyses are appropriate.

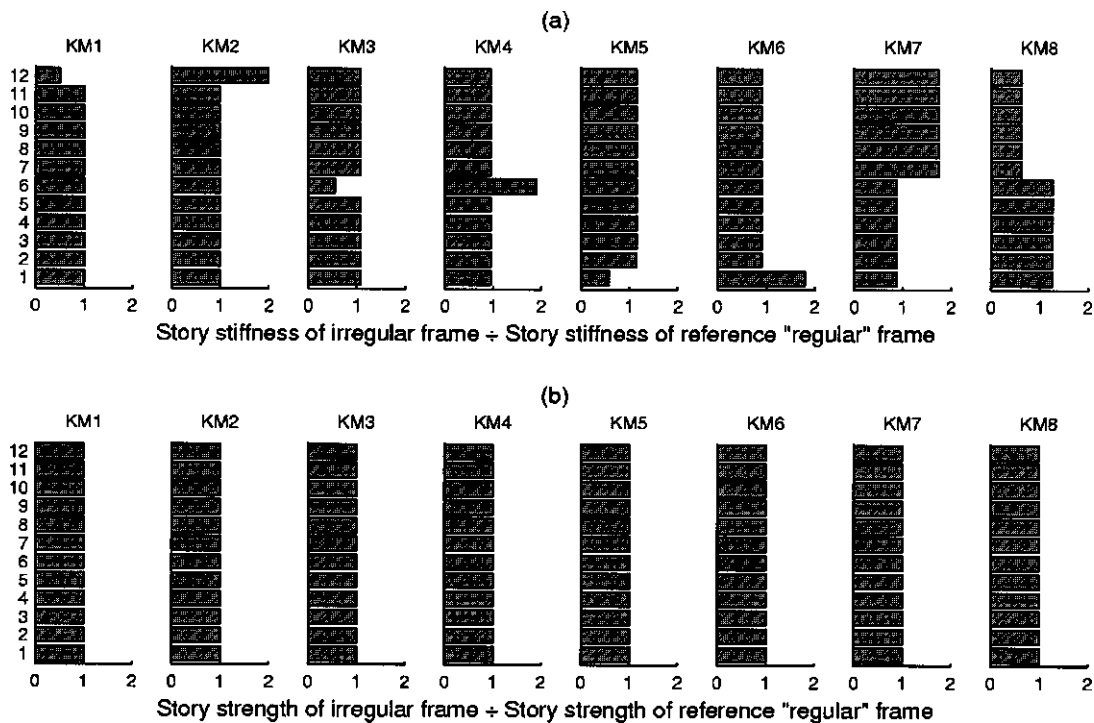
A previous study evaluated the MPA procedure for code-designed buildings (Goel and Chopra 2004). To broaden the scope and cover a wider range of building properties, this investigation studied generic frames. These frames are described fully in Chintanapakdee and Chopra (2002); a summary is included next. To investigate the influence of vertical irregularity on seismic demands for buildings and to evaluate the accuracy of MPA for irregular frames, a 12-story regular frame was defined as the reference case for comparison. The reference frame was designed so that the height-wise distribution of stiffness achieves equal drifts in all stories under the IBC lateral forces. Assuming that the second moment of cross-sectional area for each beam and its supporting columns in the story below are the same, numerical values for the flexural rigidities of structural elements were selected such that the fundamental vibration period  $T_1$  is 2.40 s. This is the value determined from  $T_U = 0.045H^{0.8}$ , where  $H$  is the total height of the frame, an equation that defines the mean-plus-one-standard deviation of measured periods of steel moment resisting frames (Goel and Chopra 1997).

The frame is designed according to the strong-column–weak-beam philosophy; therefore, plastic hinges form only at beam ends and the base of the first-story columns. The columns in other stories are assumed to remain elastic. The yield moments of plastic hinges, with bilinear (3% postyield stiffness) moment–rotation relation, are selected such that yielding occurs simultaneously at all plastic hinges under the IBC lateral force distribution. The yield base shear is  $V_{by} = (A_y/g)W$ , where  $W$  is the total weight of the frame and  $A_y$  is the median (over 20 ground motions) pseudoacceleration for a single-degree-of-freedom (SDF) system with vibration period  $T_n = T_1$  and a ductility factor  $\mu = 4$ .

The Rayleigh damping matrix is defined to obtain a damping ratio of 5% in the first and fourth modes of vibration.

### Vertically Irregular Frames

Forty-eight irregular frames, all 12-stories high, were considered to account for three types of irregularities introduced in eight different locations along the height using two modification factors, described next. Three types of irregularities in the height-wise distributions of frame properties were considered: Stiffness irregularity (*KM*), strength irregularity (*SM*), and combined-stiffness-and-strength irregularity (*KS*). Various irregular frames are obtained by modifying the stiffness or/and strength of the reference frame. To obtain a soft or stiff story, the story stiffness was divided or multiplied by a modification factor; and to obtain a weak or strong story, the story strength was divided or multiplied by a modification factor. Two values of the modification factor (*MF*) were considered:  $MF = 2$  or 5 (Chintanapakdee and Chopra 2002). For brevity, results presented here are only for  $MF = 2$ . For each of the three types of irregularities, the following eight cases were investigated: (1) Soft or/and weak top story, (2) stiff or/and strong top story, (3) soft or/and weak midheight story,



**Fig. 2.** Ratio of (a) story stiffness and of (b) story strength of stiffness-irregular frames to the corresponding properties of the regular frame for modification factor,  $MF=2$

(4) stiff or/and strong midheight story, (5) soft or/and weak first story, (6) stiff or/and strong first story, (7) soft or/and weak lower half of structure, and (8) stiff or/and strong lower half of structure.

### Stiffness-Irregular Frames

Fig. 2 shows the ratio of story stiffness, and of story strength of stiffness-irregular frames to the corresponding properties of the regular frame;  $KM_j$  denotes stiffness-irregularity case  $j$  ( $=1,2,\dots,8$ ). A total of 16 stiffness-irregular frames are considered corresponding to the 8 cases mentioned above  $MF=2$ .

The stiffness of a story was modified by changing the stiffness of the columns in that story and the beam they support. To ensure meaningful comparison of seismic demand on regular and irregular frames, their fundamental vibration period, yield base shear, and damping properties were kept the same. Modifying the stiffness of one or more stories by the  $MF$  obviously affects the vibration period. To maintain the same period as for the regular frame, all story stiffnesses were scaled uniformly, causing the ratio of story stiffnesses of irregular and regular frames to be different than the  $MF$ , as seen in Fig. 2.

The pushover curves using the IBC force distribution show that although stiffness irregularity may influence the initial slope significantly, it affects the yield strength only slightly (Chintanapakdee and Chopra 2002). All story strengths were scaled uniformly to obtain an irregular frame with the same yield base shear as the regular frame. Note that the postyield stiffness of irregular frames can be slightly different than the regular frame.

The Rayleigh damping matrix for an irregular frame is defined to maintain the modal damping ratio equal to 5% in the first and fourth modes, as for the regular frame.

### Strength-Irregular Frames

Fig. 3 shows the ratio of story stiffness and of story strength of strength-irregular frames to the corresponding properties of the

regular frame;  $SM_j$  denotes strength-irregularity case  $j$ . The story stiffnesses, fundamental period, and damping matrix of strength-irregular frames were kept the same as for the regular frame. The strength of a story was modified by changing only the strength of the beam at the top of the story (recall that the columns are assumed to remain elastic). However, in Cases 5 to 8, where the strength of the first story was modified, the strength of columns in the first story, which were designed to hinge at the base, is also changed. All story strengths are scaled uniformly to obtain an irregular frame with the same yield base shear as the regular frame, causing the ratio of the story strengths of irregular and regular frames to be different than the modification factor, as shown in Fig. 3.

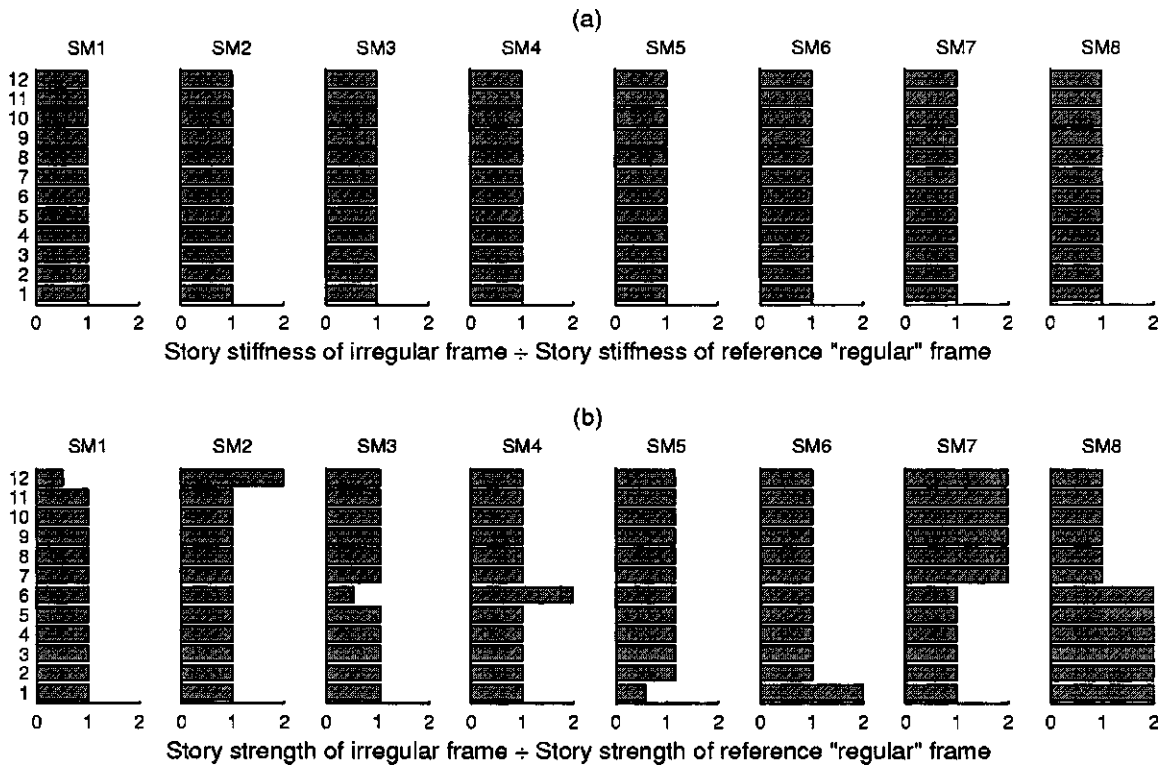
### Stiffness-and-Strength-Irregular Frames

Fig. 4 shows the ratio of story stiffness and of story strength of stiffness-and-strength-irregular frames to the corresponding properties of the regular frame;  $KS_j$  denotes combined stiffness-and-strength irregularity case  $j$ . Each frame is designed by modifying the story stiffnesses and damping matrix as described earlier for the stiffness-irregular frame, and the story strengths as described earlier for the strength-irregular frame.

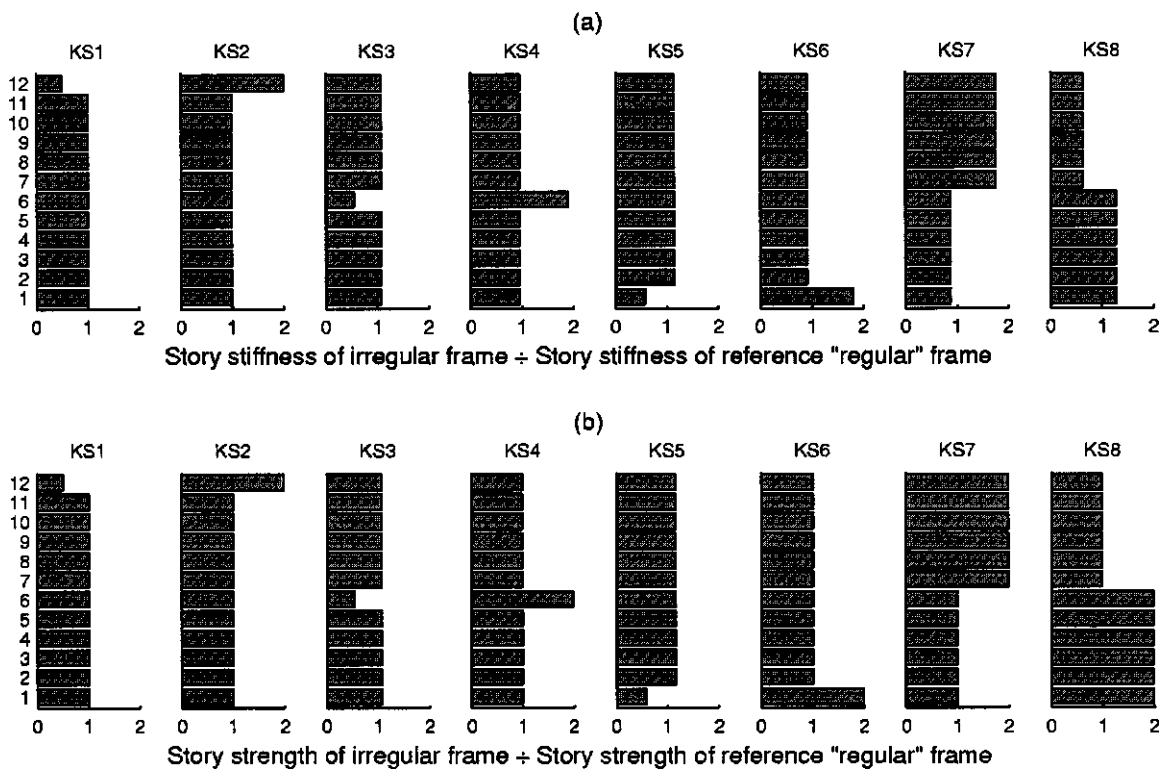
## Ground Motions and Response Statistics

### Ground Motions

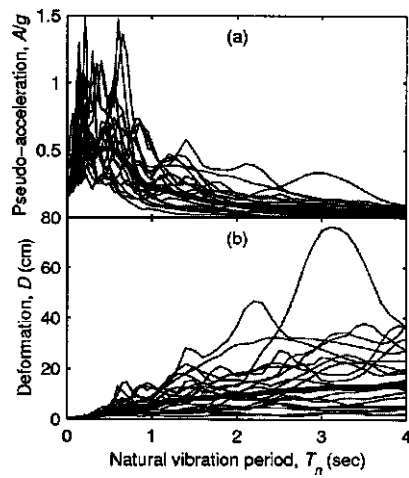
The seismic excitation for these generic frames is defined by a set of 20 large-magnitude–small-distance records (LMSR) listed in Chintanapakdee and Chopra (2002). These ground motions were obtained from California earthquakes with magnitudes ranging from 6.6 to 6.9 recorded at distances of 13 to 30 km on firm ground. Their elastic pseudoacceleration and deformation response spectra and the median spectrum are presented in Fig. 5.



**Fig. 3.** Ratio of (a) story stiffness and of (b) story strength of strength-irregular frames to the corresponding properties of the regular frame for modification factor,  $MF=2$



**Fig. 4.** Ratio of (a) story stiffness and of (b) story strength of combined-stiffness-and-strength-irregular frames to the corresponding properties of the regular frame for modification factor,  $MF=2$



**Fig. 5.** (a) Pseudoacceleration spectra and (b) deformation spectra of larger-magnitude-small-distance records set of ground motions, damping ratio=5%. The median spectrum is shown by a thicker line.

### Response Statistics

The dynamic response of each structural system to each of the 20 ground motions was determined by nonlinear RHA, and MPA (Chopra and Goel 2002) without considering  $P-\Delta$  effects due to gravity loads; details of both analysis procedures are documented in Chintanapakee and Chopra (2002). The exact peak value of structural response or demand,  $r$ , determined by nonlinear RHA (NL-RHA) is denoted by  $r_{NL-RHA}$ , and the approximate value from MPA by  $r_{MPA}$ . From these data for each ground motion, a response ratio was determined from the following equation:  $r_{MPA}^* = r_{MPA} / r_{NL-RHA}$ . An approximate method is invariably biased in the sense that the median of the response ratio differs from one, underestimates the median response if the ratio is less than one, and provides an overestimate if the ratio exceeds one.

Presented in this paper are the median values,  $\hat{x}$ , defined as the geometric mean, of  $n$  (equals 20) observed values ( $x_i$ ) of  $r_{MPA}$ ,  $r_{NL-RHA}$ , and  $r_{MPA}^*$ ; and the dispersion measure  $\delta$  of  $r_{MPA}^*$ , defined as the standard deviation of logarithm of the  $n$  observed values:

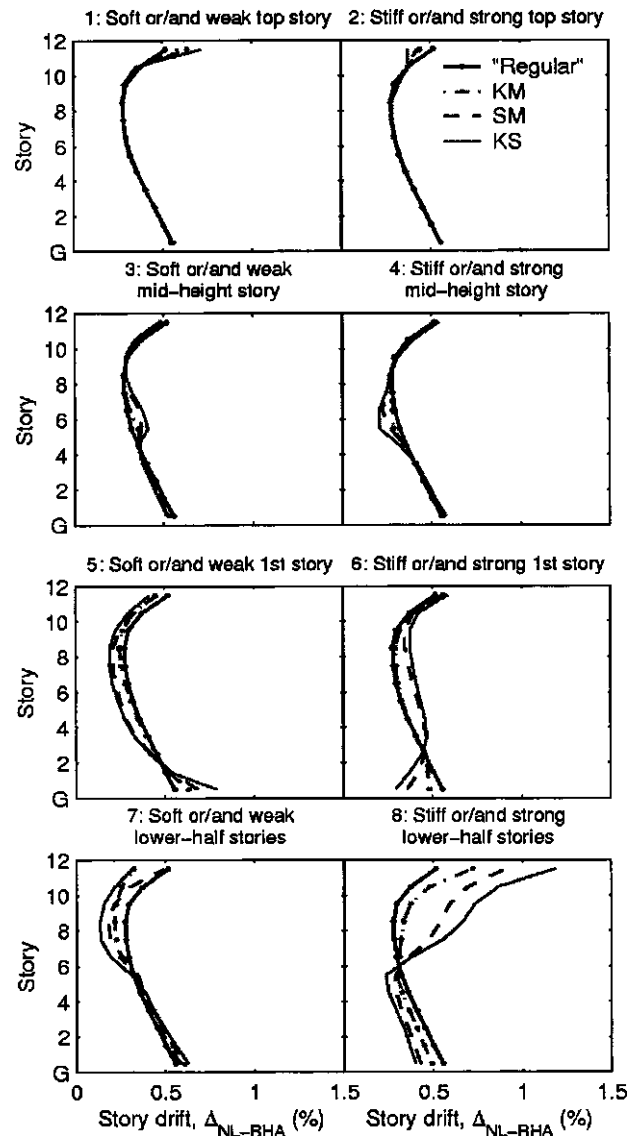
$$\hat{x} = \exp \left[ \frac{\sum_{i=1}^n \ln x_i}{n} \right] \quad (1a)$$

$$\delta = \left[ \frac{\sum_{i=1}^n (\ln x_i - \ln \hat{x})^2}{n-1} \right]^{1/2} \quad (1b)$$

For small values, e.g., 0.3 or less, the above dispersion measure is close to the coefficient of variation. This measure will be referred to as “dispersion” in subsequent sections. Eqs. 1(a) and 1(b) are logical estimators for the median and dispersion, especially if the data are sampled from lognormal distribution, an appropriate distribution for peak earthquake response of structures (Newmark and Hall 1982; Shome and Cornell 1999).

### Effect of Irregularity on Story-Drift Demands

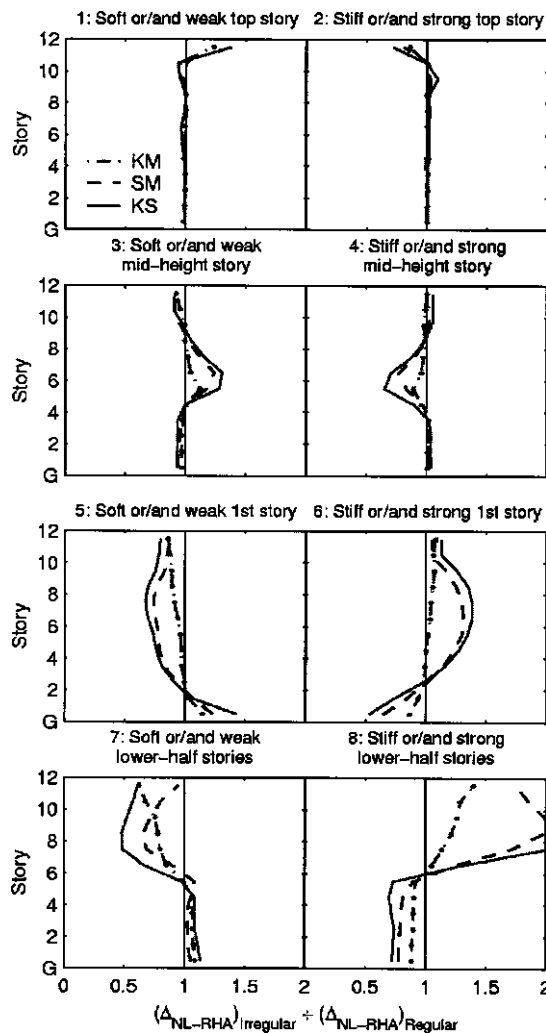
Fig. 6 presents median story-drift demands ( $\Delta_{NL-RHA}$ ) determined by NL-RHA for all cases and three types of irregularity with  $MF=2$  and compares them to the regular frame. As expected, vertical irregularity in stiffness or strength influenced the height-wise variation of story-drift demands. For each of the eight cases,



**Fig. 6.** Median story-drift demands determined by nonlinear response history analysis for regular frame and stiffness-, strength-, and combined-stiffness-and-strength-irregular frames denoted by *KM*, *SM*, and *KS*, respectively, with modification factor,  $MF=2$

the three types of irregularity influenced the height-wise variation of story drifts similarly, with the effects of strength irregularity being larger than stiffness irregularity, and the effects of combined-stiffness-and-strength irregularity being the largest among the three. This observation agrees with conclusions of Al-Ali and Krawinkler (1998).

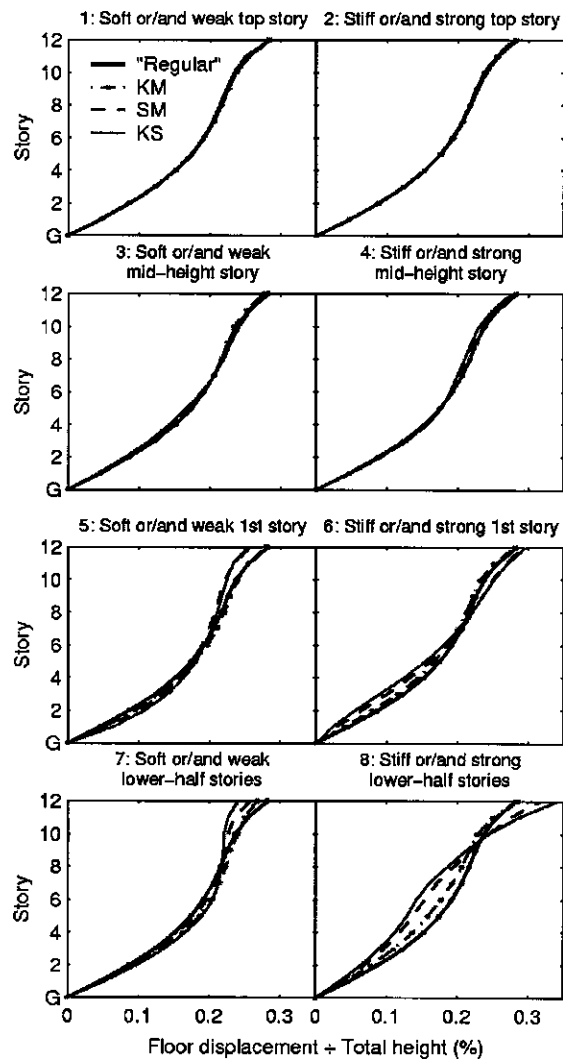
Introducing a soft and/or weak story (Fig. 6: Cases 1, 3, and 5) increases the drift demands in the modified and neighboring stories and decreases the drift demands in other stories. On the other hand, introducing a stiff and/or strong story (Fig. 6: Cases 2, 4, and 6) decreases the drift demands in the modified and neighboring stories and increases the drift demands in other stories. Cases 7 and 8 will be discussed later. These trends differ from the results reported by Al-Ali and Krawinkler (1998), where the drift demand was affected to a greater degree, but only in the soft and/or weak story; their column-hinge model restricts redistribution of seismic demands to adjacent stories.



**Fig. 7.** Ratio of the median story-drift demands of regular and irregular frames. Three types of irregularity—stiffness-, strength-, and combined-stiffness-and-strength—are included with modification factor,  $MF=2$ .

To illustrate how significantly irregularity effects drift demands, Fig. 7 presents the ratio of the median drift demands ( $\Delta_{NL-RHA}$ ) of irregular and regular frames; the difference between this ratio and unity indicates the effect of irregularity. Essentially independent of the location of the irregular story, the drift demand in a soft and/or weak story (Cases 1, 3, and 5) with  $MF=2$  increases due to stiffness ( $KM$ ), strength ( $SM$ ), and combined-stiffness-and-strength ( $KS$ ) irregularity by about 15%, 25%, and 40%, respectively. Similarly, the percentage reduction in drift demand at the stiff and/or strong story (Cases 2, 4, and 6) with  $MF=2$  was also essentially independent of the location of the irregular story. As shown in Fig. 7, however, the effect of an irregular story on drift demands at stories further away from the irregular story is strongly dependent on its location: It is significant when the irregular story is near the base (Cases 5 and 6) but almost negligible when the irregular story is near the top (Cases 1 and 2).

A soft and/or weak lower half of the frame (Case 7 in Figs. 6 and 7) slightly increases the drift demands for those stories, but significantly decreases the drift demands in stories in upper half of the frame. In contrast, a stiff and/or strong lower half of the frame (Case 8) reduces the drift demands for those stories but



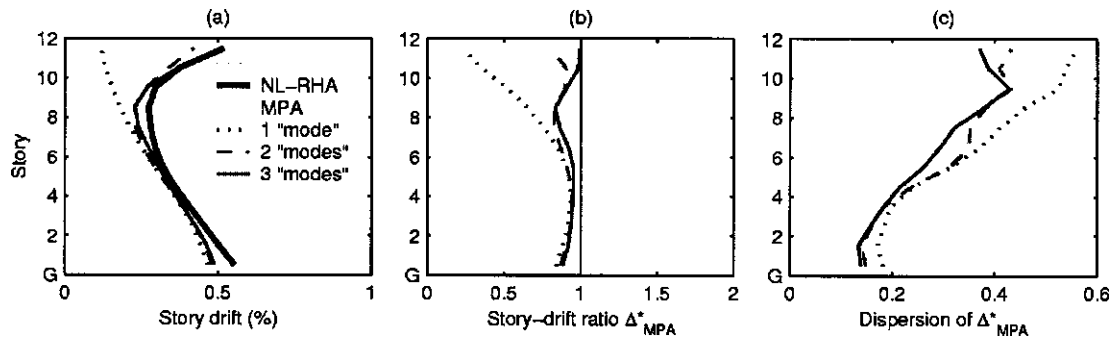
**Fig. 8.** Median floor displacements determined by nonlinear response history analysis for regular frame and stiffness-, strength-, and combined-stiffness-and-strength-irregular frames denoted by  $KM$ ,  $SM$ , and  $KS$ , respectively, with modification factor,  $MF=2$

significantly increases the drift demands in the upper half of the frame. These observations imply that drift demands occurring in the upper stories are much more sensitive to irregularity than the drift demands in the lower stories.

As expected, Fig. 6 (Cases 7 and 8) shows that the drift is amplified—relative to the regular frame—in the weaker story adjacent to the discontinuity in strength at midheight (irregularity types  $SM$  and  $KS$ ). Note this amplification is distributed over all weaker stories, not the concentrated amplification in one story as predicted by the less realistic column-hinge model (Al-Ali and Krawinkler 1998).

### Effect of Irregularity on Floor Displacements

Fig. 8 compares median values of floor displacements determined by NL-RHA for three types of irregular frames with  $MF=2$  to that of the regular frame. As long as the irregularity is in the mid- or upper stories (Cases 1–4), all three types of irregularities have very little influence on floor and roof displacements. In contrast, irregularity in the base story or lower half of the building (Cases



**Fig. 9.** (a) Median story-drift demands for the regular frame determined by modal pushover analysis with variable number of modes and by nonlinear response history analysis; (b) median story-drift ratio  $\Delta_{MPA}^*$ ; and (c) dispersion of story-drift ratios  $\Delta_{MPA}^*$

5–8) significantly influences the height-wise variation of floor displacements. As observed by Al-Ali and Krawinkler (1998), while the roof displacement is normally insensitive to vertical irregularity (Cases 1–6); however, it is significantly different for frames with a soft and weak lower half (Case *KS7*), or a stiff and strong lower half (Case *KS8*).

Although the results of Cases 5–8 in Fig. 8 may seem counterintuitive in that the roof displacements of irregular frames with a soft and/or weak first story or lower half are smaller than the regular frame, whereas the roof displacements of irregular frames with a stiff or/and strong first story or lower half are larger than the regular frames, these results can be rationalized by recognizing that the irregular frames were scaled to have the same vibration period and yield base shear as the regular frames. Thus, a frame with a soft or/and weak lower half is not softer or weaker than the regular frame in an overall sense, but its lower half is softer and/or weaker relative to the upper half, compared to the regular frame.

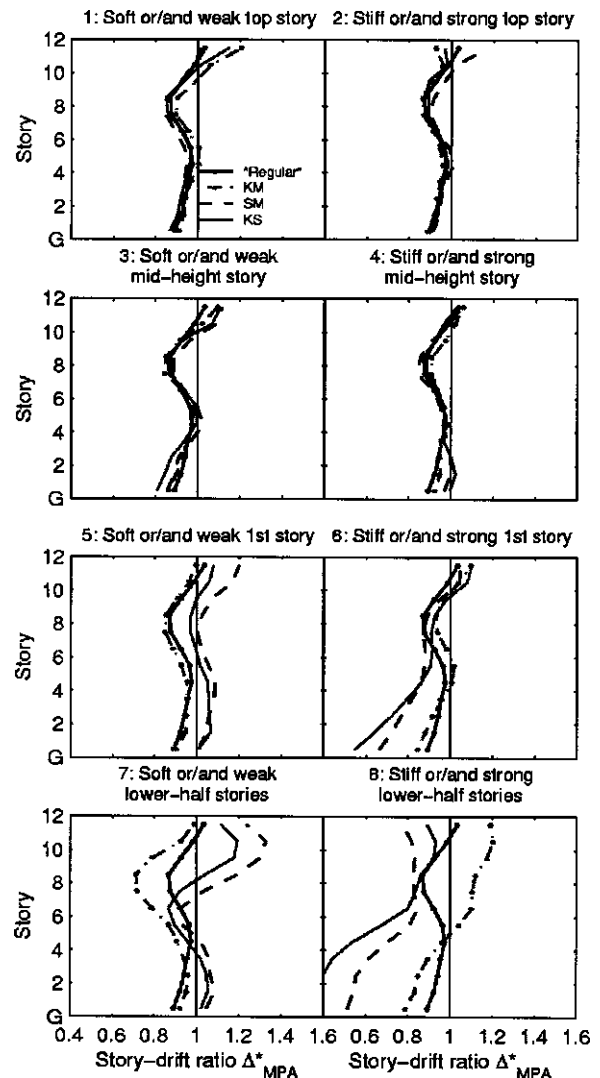
### Bias and Accuracy of Modal Pushover Analysis

The median story-drift demands determined by MPA on regular frames—including 1, 2, or 3 “modes”—are compared to the results of nonlinear RHA [Fig. 9(a)]. The MPA considering only the first mode is inadequate in estimating story drifts demands in upper stories, where contributions of higher modes are known to be significant, even in elastic systems (Chopra 2001, Chapter 18). With higher mode contributions included, however, MPA follows the height-wise variation of drift demands reasonably well. Examining the median and dispersion of the ratio  $\Delta_{MPA}^*$  indicates that the bias in MPA [Fig. 9(b)] and its dispersion [Fig. 9(c)] decreases as higher modes are included; however, even with three modes included, MPA tends to underestimate the drift demands in all stories of this frame except the top one. This underestimation is similar to a trend found in response spectrum analysis of elastic frames and does not disappear even if all modes are included. In brief, MPA underestimates or overestimates the demand depending on the height, vibration period, and design ductility (Chintanapakdee and Chopra 2003).

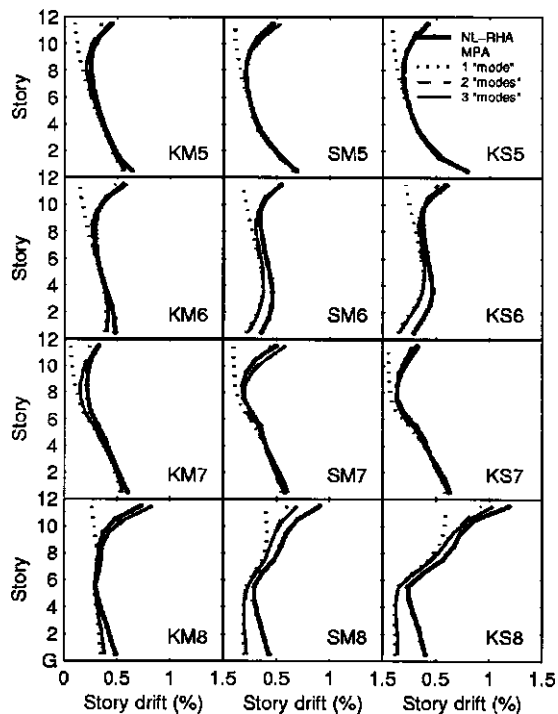
Fig. 10 presents the median of the ratio  $\Delta_{MPA}^*$  of drift demands obtained from MPA (including four modes) and from NL-RHA for all eight cases and three types of irregularity with  $MF=2$  and compares them to results for the regular frame. As mentioned earlier, the difference between median  $\Delta_{MPA}^*$  and unity represents the bias in the MPA procedure. These results demonstrate that the bias in the MPA procedure does not increase, i.e., its accuracy does not deteriorate, in spite of irregularity in stiffness, strength,

or both stiffness and strength, provided the irregularity is in the top story or midheight story (Cases 1–4).

The MPA procedure is less accurate for structures where the irregularity is in the first story or in the lower half of the frame (Cases 5–8). The bias is significantly larger for: (1) The lower stories of an irregular frame with strong or stiff-and-strong first



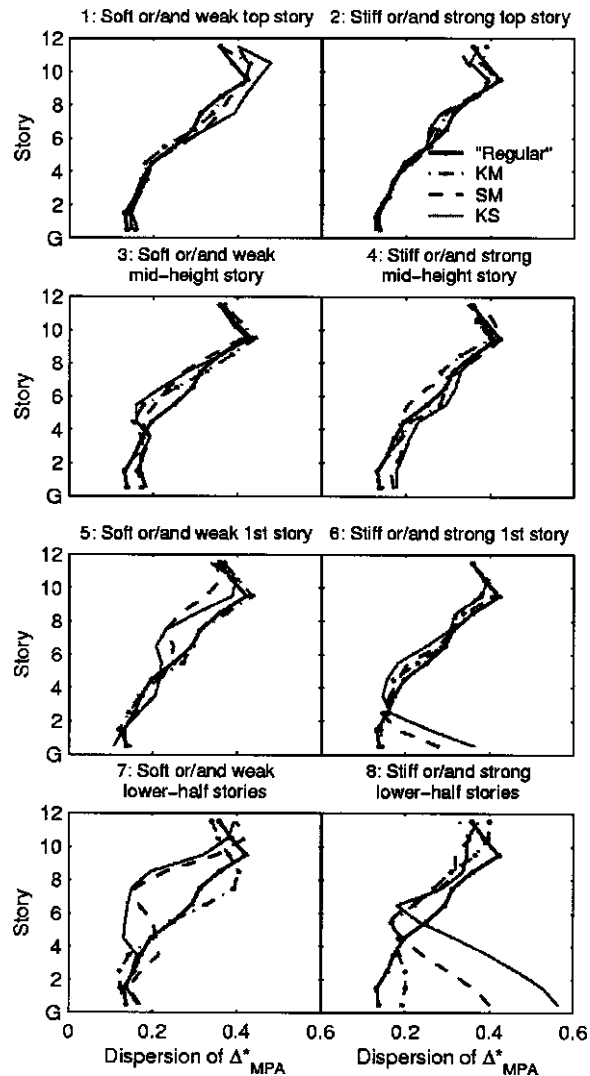
**Fig. 10.** Median story-drift ratios  $\Delta_{MPA}^*$  for regular frame and stiffness-, strength-, and combined-stiffness-and-strength-irregular frames with modification factor,  $MF=2$



**Fig. 11.** Median story-drift demands of stiffness-, strength-, and stiffness-and-strength-irregular frame Cases 5–8 with modification factor,  $MF=2$ , determined by MPA including 1, 2, and 3 modes and by nonlinear response history analysis

story (Cases *SM6* and *KS6*) compared to the regular frame, (2) the upper stories of frame with a soft, weak, or soft-and-weak lower half (Cases *KM7*, *SM7*, *KS7*), and (3) the lower stories of a frame with stiff, strong, or stiff-and-strong lower half (Cases *KM8*, *SM8*, *KS8*). Note that the bias in the MPA procedure for frames with a soft, weak, or soft-and-weak first story (Cases *KM5*, *SM5*, *KS5*) is about the same as found for the regular frame.

The larger bias in the MPA estimates suggests that the MPA procedure is not appropriate for estimating story drift demands for irregular frames corresponding to Cases 6–8. To better understand the limitations of the MPA procedure, Fig. 11 compares the median story drift demands determined by the MPA including three modes to the results of NL-RHA. The MPA procedure detects the concentration of drift demand in lower stories due to the presence of a weak and/or soft first story (Case 5) and estimates reasonably well the drift demands in all stories. Similarly, the MPA procedure identifies the larger drift demands in lower stories of frames with weak and/or soft lower half (Case 7); the drift estimates are accurate enough for practical application, although they are less accurate compared to the regular frame. Although the MPA procedure detects the larger drifts in the upper stories of frames with a stiff and/or strong first story (Case 6), it may underestimate significantly the smaller drift demands in the first few stories of the frame. Although this underestimation by the MPA procedure becomes larger in frames with stiff and/or strong lower half (Case 8), MPA still detects larger drifts in the upper stories of such frames and estimates them to a useful degree of accuracy. The overall impression that emerges is that the MPA procedure is able to identify the stories with the largest drifts, even for Cases 5–8, and hence able to detect critical stories in such frames with the caveat that it may underestimate significantly the smaller drift demands in other stories.



**Fig. 12.** Dispersion of story-drift ratios  $\Delta^*_{MPA}$  for regular frame and stiffness-, strength-, and combined-stiffness-and-strength-irregular frames with modification factor,  $MF=2$

Also presented in Fig. 11 are the drift demands computed by MPA considering only the first mode of vibration. Clearly, even for irregular frames, the first mode alone is inadequate in estimating the drift demands in the upper stories where the contributions of higher modes are significant.

### Dispersion of Modal Pushover Analysis

Fig. 12 presents the dispersion of  $\Delta^*_{MPA}$  for all eight cases and three types of irregularity with  $MF=2$  and compares them to the regular frame. The dispersion of  $\Delta^*_{MPA}$  for irregular frames is similar to that found for the regular frame, except that it is much larger for the lower stories of Cases 6 and 8 of strength- and stiffness-and-strength-irregular frames, for which the bias in MPA is also large (Fig. 10). Except for these two cases, the MPA procedure should be similarly reliable in estimating the seismic demands of irregular and regular frames due to an individual ground motion, although pushover analysis procedures may be inappropriate for such applications.



## Conclusions

This investigation of the effects of stiffness, strength, and combined stiffness-and-strength irregularity on seismic demands of strong-column–weak-beam frames (beam-hinge model) has led to the following conclusions:

1. The three types of irregularities similarly influence the height-wise variation of story drifts, with the effects of strength irregularity being larger than stiffness irregularity, and the effects of combined-stiffness-and-strength irregularity being the largest among the three.
2. Introducing a soft and/or weak story (Cases 1, 3, and 5) increases the story drift demands in the modified and neighboring stories and decreases the drift demands in other stories. On the other hand, a stiff and/or strong story (Cases 2, 4, and 6) decreases the drift demand in the modified and neighboring stories and increases the drift demands in other stories.
3. Drift demands in the upper stories are much more sensitive to irregularities in the lower stories than the response of lower stories is affected by irregularities in the upper stories.
4. While the roof displacement is usually insensitive to vertical irregularity (Cases 1–6), it is significantly different for frames that are stiffness-and-strength irregular in their lower half (Cases *KS7* and *KS8*). Irregularity in the base story or lower stories (Cases 5–8) has a significant influence on the height-wise distribution of floor displacements.

Concerning the accuracy of the MPA procedure in estimating seismic demands for vertically irregular frames, the following conclusions were reached:

1. The bias in the MPA procedure does not increase, i.e., its accuracy does not deteriorate, in spite of irregularity in stiffness, strength, or stiffness and strength provided the irregularity is in the top story or midheight story (Cases 1–4).
2. The MPA procedure can be more biased, i.e., less accurate, relative to the regular frame in estimating the seismic demands of frames with strong or stiff-and-strong first story; soft, weak, or soft-and-weak lower half; stiff, strong, or stiff-and-strong lower half. In contrast, the bias in the MPA procedure for frames with soft, weak, or soft-and-weak first story is about the same as for the regular frame.
3. In spite of the larger bias in estimating drift demands for some stories in Cases 6–8, the MPA procedure identifies the stories with largest drift demands and estimates them well, detecting the critical stories in such frames.
4. The dispersion of  $\Delta_{MPA}^*$  for irregular frames is similar to the regular frame, except in the lower stories of the frames with a strong first story (Cases *SM6* and *KS6*) or with strong lower half (Cases *SM8* and *KS8*), for which the bias in MPA is also large.
5. The MPA procedure also provides usefully accurate seismic demands for irregular frames, except for those with a strong first story or strong lower half. The seismic demands for such irregular frames should be determined by NL-RHA.

The preceding conclusions were based on an investigation of frames designed according to the strong-column–weak-beam philosophy, pervasive, and preferable in seismic design. These conclusions may not be valid if columns are also expected to yield. The conclusions are also limited to frames with 12 or fewer stories.

## Acknowledgments

This research investigation was funded by the National Science Foundation under Grant No. CMS-9812531, a part of the U.S.–Japan Cooperative Research in Urban Earthquake Disaster Mitigation. This financial support is gratefully acknowledged. Our research has benefitted from discussions with Professor Helmut Krawinkler of Stanford University, who also provided the LMSR ground motion ensemble.

## References

- Al-Ali, A. A. K., and Krawinkler, H. (1998). “Effects of vertical irregularities on seismic behavior of building structures.” *Rep. No. 130*, John A. Blume Earthquake Engineering Center, Stanford Univ., Stanford, Calif., 198.
- Building Seismic Safety Council (BSSC). (1997). “NEHRP Guidelines for the Seismic Rehabilitation of Buildings.” *FEMA-273*, Federal Emergency Management Agency, Washington, D.C.
- Chintanapakdee, C., and Chopra, A. K. (2002). “Evaluation of the modal pushover procedure using vertically ‘regular’ and irregular generic frames.” *Rep. No. EERC 2003-03*, Earthquake Engineering Research Center, Univ. of California at Berkeley, Berkeley, Calif., 224.
- Chintanapakdee, C., and Chopra, A. K. (2003). “Evaluation of modal pushover analysis using generic frames.” *Earthquake Eng. Struct. Dyn.*, 32(3), 417–442.
- Chopra, A. K. (2001). *Dynamics of structures: Theory and applications to earthquake engineering*, 2nd Ed., Prentice-Hall, Englewood Cliffs, N.J., 844.
- Chopra, A. K., and Goel, R. K. (2002). “A modal pushover analysis procedure for estimating seismic demands for buildings.” *Earthquake Eng. Struct. Dyn.*, 31(3), 561–582.
- Goel, R. K., and Chopra, A. K. (1997). “Period formulas for moment-resisting frame buildings.” *J. Struct. Eng.*, 123(11), 1454–1461.
- Goel, R. K., and Chopra, A. K. (2004). “Evaluation of modal and FEMA pushover analyses: SAC buildings.” *Earthquake Spectra*, 20(1), 225–254.
- International Code Council. (2000). *International Building Code*, Falls Church, Va.
- Newmark, N., and Hall, W. J. (1982). “Earthquake spectra and design.” *Earthquake Engineering Research Institute*, Berkeley, Calif., 103.
- Shome, N., and Cornell, C. A. (1999). “Probabilistic seismic demand analysis of nonlinear structures.” *Rep. No. RMS-35*, Dept. of Civil Engineering, Stanford Univ., Stanford, Calif., 320.
- Valmundsson, E. V., and Nau, J. M. (1997). “Seismic response of building frames with vertical structural irregularities.” *J. Struct. Eng.*, 123(1), 30–41.