Construction and Building Materials 24 (2010) 1449–1455

Contents lists available at ScienceDirect

Construction and Building Materials

journal homepage: www.elsevier.com/locate/conbuildmat

Application of genetic algorithm for modeling of dense packing of concrete aggregates

Konstantin Sobolev [a,*], Adil Amirjanov [b]

[a] Department of Civil Engineering and Mechanics, College of Engineering & Applied Science, University of Wisconsin-Milwaukee (UWM), USA
[b] Department of Computer Engineering, Near East University, Nicosia, N. Cyprus, USA

ARTICLE INFO

Article history:
Received 26 May 2009
Received in revised form 15 January 2010
Accepted 15 January 2010
Available online 20 February 2010

Keywords:
Genetic algorithm
Sequential Packing Algorithm
Simulation model
Packing density
Aggregates
Particle size distribution
Concrete mixture proportioning

ABSTRACT

Sequential Packing Algorithm (SPA) was developed to model the dense packing of large assemblies of particulate materials (in the order of millions). These assemblies represent the real aggregate systems of portland cement or asphalt concrete. To improve the SPA performance, the program engine was updated with a genetic algorithm (GA) search module. Multi-cell packing procedures, fine adjustment of the algorithm's parameters, as well as implementation of GA were effective tools to optimize the computational resources, to speed-up the SPA and to pack very large volumes of spherical entities.

The developed algorithm generates and visualizes dense packings corresponding to concrete aggregates. The influence of model variables on the degree of packing and the corresponding distribution of particles was analyzed. Based on the simulation results, different particle size distributions of particulate materials are correlated to their packing degree. These packings agreed well with the standard requirements and available research data. The results of the research can be applied to the optimal proportioning of concrete mixtures.

Published by Elsevier Ltd.

1. Introduction

The properties and behavior of particulate composite materials, such as portland cement and asphalt concrete mixtures, depends to a large extent on the properties of their main constituent – the aggregates [1–5]. Among the most important parameters affecting performance of concrete are packing density, compaction degree, and the corresponding particle size distribution (PSD) of aggregates. Better aggregate packing improves the main engineering properties of concrete: strength, modulus of elasticity, creep, and shrinkage. Further, such packing reduces the volume of binder, thereby providing significant cost savings. Very early reports on concrete technology have already emphasized the important effects of aggregate grading on concrete performance [1,4,5]. Recently, the problem of the best-possible proportioning of aggregates and their contribution to optimal concrete mix has been the subject of many experimental and theoretical investigations [1–13]. Nevertheless, better understanding of the problem of densely packing large assemblies of particulate materials of non-uniform sizes needs further attention.

Several reports discuss the important contribution of the shape of the particle on packing [4,8,12]; however, it is considered acceptable to model and represent natural or artificial aggregates (which are mainly irregular in shape) by using groups of spheres of different diameters. This assumption partially reflects the existing methods of particle size analysis and helps to simplify packing calculations [4,6,13]. The packing density of spheres is characterized either by the packing fraction η, or by porosity $\varepsilon = 1 - \eta$, which is a fraction of the unoccupied volume. As Kepler conjectured – and Hales proved [1,2] – the optimal packing of equal hard spheres is the face-centered cubic (fcc) arrangement with a maximal density of $\pi/\sqrt{18} \approx 0.7405$; however, the maximum packing value of randomly packed, equally-sized spheres in packing experiments was only 0.64 [14]. An increase in density could be achieved only if spheres of different sizes are used. In this case, smaller spheres must fit into the cavities between the already-packed larger neighbors – a procedure similar to the arrangement of small spheres in the Apollonian problem [5,15]. By using a wide range and large number of particles, the packing density can theoretically closely approach unity.

The first attempts to provide the "best" optimal particle size distribution were based on trials with balls of different diameters [4,5,16–19]. These experiments resulted in the aggregate's distribution curves, which are currently accepted as industry standards. One of the early examples presented by Fuller–Thompson–Andreasson [5] is a series of curves that are widely used for the optimization of concrete and asphalt aggregates:

$$P_i = 100\left(\frac{d_i}{D_{\max}}\right)^{\psi}$$

* Corresponding author.
E-mail address: sobolev@uwm.edu (K. Sobolev).

0950-0618/$ - see front matter Published by Elsevier Ltd.
doi:10.1016/j.conbuildmat.2010.01.010

where P_i is the total percent of particle passing through (or finer than) sieve; D_{max} the maximal size of aggregate; d_i the diameter of the current sieve; ψ is the exponent of the equation (0.45–0.7).

Because it is relatively simple to achieve the "target" distribution with a minimum deviation using a few (at least two) sets of particulate materials [7,20], this optimal distribution method is extensively used in practice. Yet, despite its usefulness, this method cannot predict the packing degree and spatial arrangement of particles of the resultant mixture.

Due to its practical importance, the packing problem of real systems still remains a challenging subject that attracts the attention of many scientists. A model developed by Aim and Toufar has provided a useful tool for explaining the packing mechanism [19,21]. Their model focuses on packing two groups of equally sized (where each group has a different size) spheres. Each group is represented by the spheres' characteristic diameter and eigenpacking degree. After minor adjustments, this model provides a good explanation of the experimental results related to concrete aggregates [2]. A similar model was proposed by de Larrard for application in concrete technology [3]. To apply this model, it is necessary to measure or estimate the eigenpacking degree of the individual group of particles. Moreover, the real particulate systems are usually better represented by the particle size distributions rather than by the characteristic diameter. These two constraints limit the applicability of this model.

With the advancement of computers, the packing problems of real particulate systems have become a challenging subject for engineers [4,6,13–15,22–27]. Advanced algorithms are usually based on the movement of particles (represented as spheres or ellipsoids) due to rolling or sliding under the compaction gradient. Based on this strategy, the particles in a rigid container are forced to occupy the best vacant positions within the neighborhood. Modern modeling approaches include better insight into the natural packing process and even full-scale modeling of the compaction of particulate systems [23–27]. Additional factors (such as friction and deformation) acting at the contact points were found to be essential for modeling the dynamic processes involving the particulate materials [26,27]. To realize such algorithms, each movement of a particle requires the solution of the corresponding differential equations. A comprehensive survey of packing algorithms was recently presented by Jia and Williams [22].

An interesting approach to dense packing was suggested by Anishchik and Medvedev [15] involving the solution of the three-dimensional Apollonian problem using Voronoi–Delaunay method extended to deal with non-equal spheres. Based on this method, a new sphere is packed into the Voronoi S-region (the region of a volume, all points of which are closer to the surface of given sphere than to the surfaces of other spheres in the packing). Using this approach, a very high packing degree of 90% was achieved using a relatively small number of particles (about 40,000) [15].

It is believed that optimal particle size distribution (PSD) corresponds to the "best" or the densest packing of the constituent particles; however, modeling the packing of large particulate assemblies had demonstrated that the densest arrangements (PSD) of particles are actually not realized in concrete technology [13]. As it was shown, only "gap-gradings" could be considered to some extent as a sort of dense arrangement of particles. The majority of "practical" concrete aggregates gradings lying between the 0.45–0.7 power curves (i.e., Fuller curves with $\psi = 0.45–0.7$) are actually "Loose-Initially-Packed", LIP systems [13]. The fundamental characteristic of such systems is related to a wide range of particle sizes required to achieve the high packing degree at a limited number of largely-sized particles. These are opposed to almost "perfect" geometric, regular arrangements of high density, realized with a relatively narrow size range.

Despite recent progress in the development of packing algorithms, it is evident that a new approach is needed to model the packing of large assemblies of particulate materials (up to 10 million particles) representing the aggregate structure of portland cement concrete. On the one hand, the computer simulation algorithm must imitate the natural packing processes; on the other hand, the developed approach must be easily applicable to solve practical problems. Moreover, the packing process must be accomplished in a reasonable period of time and, preferably, using conventionally available computational recourses.

This study proposes the enhancement of SPA with a genetic algorithm (GA) module. A GA can be used to search the free space to inscribe the maximum-sized spheres among the previously packed spheres. Only few articles deal with the sphere-packing problem using GA. For example, Franck-Oberaspach et al. employed a GA for the solution of a two-dimensional packing problem of different rigid objects [14]. In their work an arbitrary number of points is arranged within a given two-dimensional connected region in a such way that their mutual distances and the distance from the region boundary reaches a maximum [14]. Cornforth applied a GA for the placement of overlapping grids for the input space quantization in machine-learning algorithms, which is often visualized in a three-dimensional space with the task of fitting the maximum number of oranges (represented by equal spheres) into a box [15]. Therefore, Cornforth applied a GA for a sphere-packing problem to effectively achieve the maximum density of equal spheres in a given space [15]; however, the prior art implementations were not used to model the packing of large particulate composites.

2. Description of the packing model

It was found that a good approximation of particulate systems of elementary volume can be achieved within a container with a rigid or periodic boundary when the center of the particle is randomly located at the grid of a cubic lattice [13]. In this case, a thick, 3D mesh with an opening size of less than 1/100 of the minimal diameter of the particle must be used to minimize any possible computational error. In the proposed model, the particle is considered as a discrete element, which is represented by a sphere. In this work, the packing into a container with periodic boundaries was used; this was considered to represent the elementary volume of particulate assemblies and to eliminate the wall effect. The simulation was realized in a cube C(1), where 1 is a length of side and at a size of the lattice grid of 1/32766.

A two-dimensional representation of Sequential Packing Algorithm (SPA) is shown in Fig. 1. New spherical particles are sequentially placed into the cube with the center glued to the node of a lattice grid and with radius in the range of $r_{min} < r \leqslant r_{max}$. The radius r_{max} is fixed experimentally, but r_{min} is decreased gradually, thereby allowing larger spheres to be placed in a cube prior to the placement of smaller ones. The formula for calculating r_{min} is as follows:

$$r_{\min(n)} = \frac{r_{max}}{(k_{red})^N}$$

where $k_{red} = 1 + 10^K$ is a constant for reduction the r_{min}, K is the reduction coefficient and N is a number of the packing attempts (steps) required to gradually reduce r_{min}.

Initially, the cube is pre-packed with an initial sphere (or spheres). Fig. 1 shows a randomly pre-packed sphere SP1. Then, the center of a new sphere is randomly generated within the cube lattice. Before locating the sphere with radius r, the various conditions are examined:

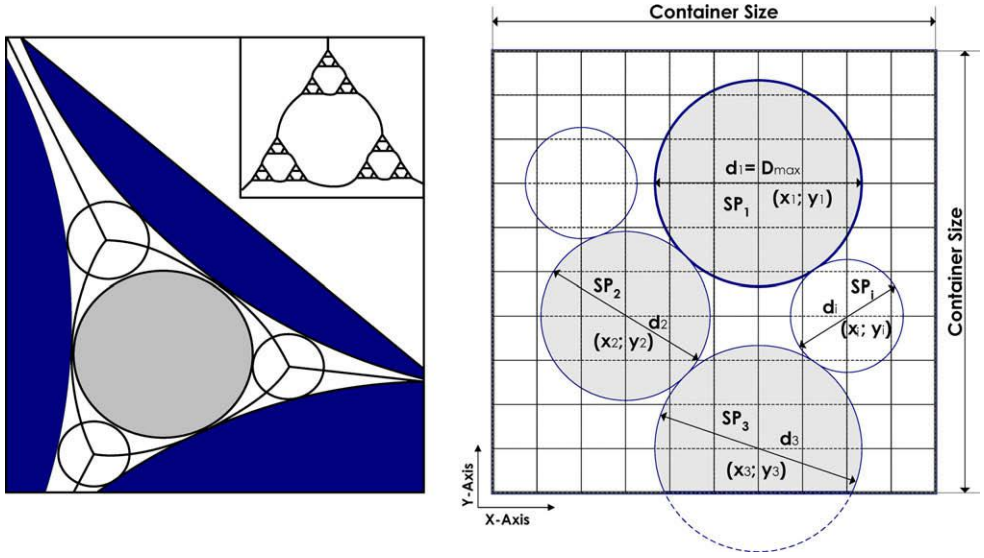

Fig. 1. 2D illustration of packing problem: (a) apollonian problem: inscribing a new particle so a new particle has only three geometrical neighbors or tree faces of the Voronoi "S" region (for 3D it has four faces), *after* [15]; (b) the Sequential Packing Algorithm, SPA, after [13].

- The center of the new sphere cannot be located inside of any already packed spheres.
- New sphere cannot cross any already packed sphere (no overlapping of spheres is permitted).
- The minimum distance to the surface of any already packed sphere should be greater than r_{min}.

The described SPM procedure results in a highly dense packing arrangement [13]. Fig. 1 shows the spheres SP_2, SP_3, and SP_i are placed in the case if the d_2, d_3, and d_i, respectively, satisfy the conditions above.

The packing of a cube is terminated as all predetermined spheres are packed. Commencing the packing process of each sphere, the volume fraction of solid particles, V is calculated:

$$V = \sum_{i=1}^{N_{total}} V_i$$

where V_i is a volume of a particular sphere and N_{total} is the number of the spheres packed. Based on the individual volumes of newly packed spheres, the particle size distribution of particles and packing degree are updated. It can be seen that the Sequential Packing Algorithm (SPA) can be considered as a particular case of the Particle Suspension Model (PSM) [28]; the detailed description of SPA routine (including multi-cell mode) is reported somewhere [29].

2.1. Application of the genetic algorithm (GA) for packing spheres

Genetic algorithm (GA) is one of the computational methods based on an analogy to natural evolution and an example of a biologically inspired computation. GA is a population-based computational method in which one individual of the population undergoes a simulated evolution [30–32]. When a GA is applied to solve a specific problem, these individuals normally represent potential solutions to the problem. GA includes a selection method in which individuals (represented by a data structure) that are more fit are more likely to survive. The fitness criteria may be defined in terms of how well the individual (i.e., data structure) solves the problem. GA includes some variation-generating methods in which individuals can generate new individuals that different from existing ones. A mutation and a recombination are two common variation-generating methods in a GA. These methods have been found effective for solving a wide range of global numerical optimization problems [30]. The advantage of a GA is that it does not require consideration of the landscape of a search space, nor the shape of an optimized function [30]; thus, a GA is a universal tool for a number of optimization problems.

To realize a GA for an arrangement of a new sphere with maximum radius within the available free space, a population of solutions (N_{pop}), that is, the spheres with various radii, is initially generated. Every sphere is represented by a binary string of length L containing the coordinates of the center and the radius of the sphere. Next, the pair of spheres is selected randomly as parents to produce the new spheres (children) for the next generation. This reproduction is accomplished by a crossover operation and by a mutation operation. A crossover operation partially exchanges the binary code between two parents with a probability p_c, but a mutation operation is applied to change the bit position of a binary string from 0 to 1, or vice–versa, with a probability p_m. New spheres are then evaluated by a "fitness function" (an objective function), in such a way that only the best spheres "survive" and "generate" a next population of spheres (N_{pop}). This procedure is applied for a certain number of generations (N_{gen}), at the end of which the best sphere (i.e., with maximum radius) is packed within the free available space. The quality of a solution with a reasonable computational cost that is directly proportional to a number of generations is usually a trade-off [31]. For SPA–GA, the constraint-handling method proposed by Amirjanov [32] was selected. This method, "Changing Range GA" (CRGA), adaptively shifts and shrinks the search space by employing feasible (with satisfied constraints) and infeasible (with unsatisfied constraints) spheres in the population to find a sphere with maximum radius (a global maximum). CRGA significantly improves the speed of convergence to the global maximum with reasonable precision [32]. According to the method, an elite subset (h_s) of ranked individuals (spheres) from a whole population (N_{pop}) is selected in every generation. Individuals are ranked according to their objective function by means of a stochastic ranking procedure [33], which maintains the balance between preserving the feasible individuals (with satisfied constraints) and rejecting infeasible ones (with unsatisfied constraints). This balance is derived from a stochastic bubble-sort algorithm in which a probability parameter P_f was introduced to compare any pair of adjacent spheres to determine a better one [33]. Next, the elite subset of spheres is used to assess the coordi-

nates of a center of attraction (or reference point), which are identified by calculating the mean of coordinates of the centers of the spheres from the elite subset [32]. Finally, the size of the search space is shrunk relative to the previous size using coefficient k_r and shifts to the center of attraction. The shrinking of the search space continues within every generation until the size of the region becomes less or equal to t_r of an initial size of the search space. The shrinking and the shifting mechanisms concentrates the search space to a certain cell of a cube. Consequently, the number of evaluations required to examine the set of constraints (overlapping spheres) is significantly reduced [32].

3. Research program

In this research program, the packing into a container with periodic boundaries was considered in order to represent the elementary volume of particulate composite and to eliminate the wall effect. The ratio of the container size to the maximum diameter of the sphere was fixed at 3.3 (that is a common assumption related to the density measurements when the wall effect is eliminated). The total amount of spheres used in the packing trials (N_{total}) was one million (1 M) and the mesh size (pixel) was 1/32766 of the container length. The research program considered the investigation of SPA with following parameters: (1) the reduction coefficient was varied from −1 to +1 where higher values provided a very quick drop in the minimal size of the particles and resulted in the formation of loose or "diluted" packing arrangements; and (2) since the effect of the number of packing trials per cycle, N on the packing arrangements obtained with SPA was not investigated previously, in this research program N was varied from 10^4 to 10^6. The variable parameters and their levels for the conducted experiments are presented in Table 1.

The details of GA routine and parameters were defined and refined by extensive experimental program as reported by Amirjanov [32]. The following values of GA parameters were established using the trial runs for the best performance of GA to simulate SPA–GA mode of packing: $N_{pop} = 50$, $p_c = 0.85$, $p_m = 0.02$, $L = 15$ bits, $P_f = 0.42$, $h_s = 0.2$, $k_r = 0.99$, $t_r = 0.0025$, $N_{gen} = 150$ [32]. The best performance of GA intends to provide a minimum time (a computational cost) to reach a reasonably high quality solution. The parameters N_{pop} and N_{gen} which have the most influence on the computational cost were established by the preliminary experiments. To assess the quality of the GA the latter was applied to solve the optimization problem with a known solution. In these preliminary experiments, a rather simple optimization problem was constructed; that is, to inscribe a single sphere with a known radius and known coordinates of its centre between initially prepacked spheres. In this experiment after 150 generations, the GA found the coordinates of the centre with an error less than 0.2%; for example, after 250 generations the error was reduced to 0.1%. In order to reduce the computational cost the first option was selected for the main experiment. The values of parameters p_c and p_m were selected from the range established in [30]. The parameters L, h_s, k_r and t_r also influence the quality of the solution and their optimal values were adopted from [32]. The value of parameter P_f was set according to [33].

Table 1
The experimental matrix used for evaluation of the simulation model.

Total amount of spheres, N_{total}	Number of packing trials, N	Reduction coefficient, K
1 M = 10^6	10 k = 10^4	−1
	100 k = 10^5	0
	1 M = 10^6	+1

4. Results and discussion

The results of the simulation algorithm are presented in Fig. 2 and Table 2, demonstrating the generated particle size distributions (PSD) with the passing values given for specific particle sizes (that are standard for the sieve analysis and determined by a formula: $d_i = D_{max}/2^m$, where D_{max} is the maximal size of the sphere and $m = 0, 1, \ldots, k$). Fig. 3 provides the visualization of the packing patterns obtained with 1000 spheres at different reduction coefficients.

4.1. Experimental results and discussion

4.1.1. Implementation of SPA and SPA–GA to modeling particle size distribution

The major interest of the model application is seen in the development of the distribution curves that match those used in concrete technology (Figs. 2 and 4). In the case of SPA implementation, the best (the most dense) gradings with 80–90% packing degree are obtained with the reduction coefficient of 0 and less and a high value of number of packing trials, N of 10^5

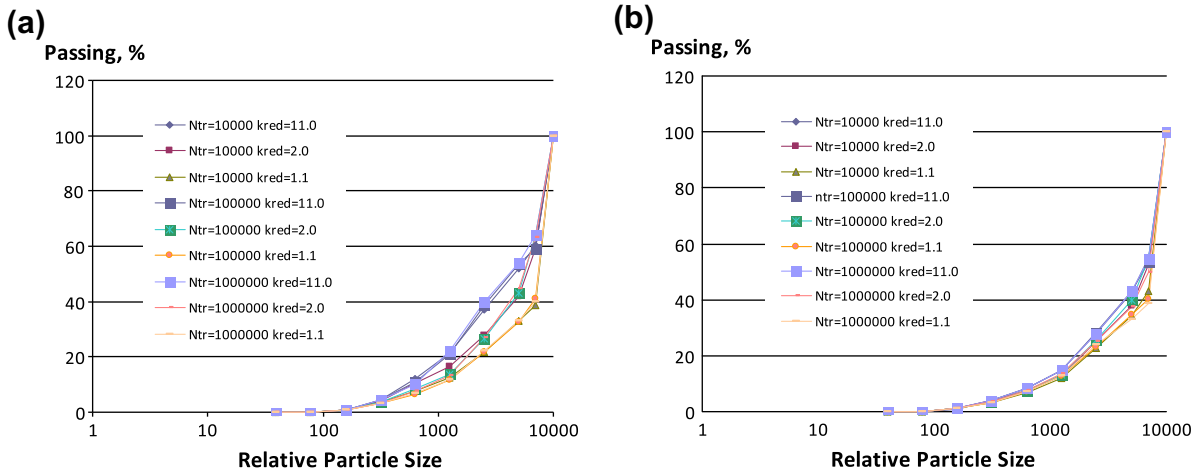

Fig. 2. The particle size distribution corresponding to: (a) SPA and (b) SPA–GA experiments (N_{tr} is a number of the packing attempts required to gradually reduce minimal radius boundary).

Table 2
Comparison of the performance of SPA and SPA–GA in packing experiments.

	N_{trials} = 10,000			N_{trials} = 100,000			N_{trials} = 1,000,000		
K	+1	0	−1	+1	0	−1	+1	0	−1
k_{red}	11	2	1.1	11	2	1.1	11	2	1.1
SPA	74.88	78.47	83.29	75.8	80.54	85.68	76.44	82.94	88.61
SPA–GA	91.49	91.94	92.95	91.45	92.06	93.61	91.49	92.23	93.87

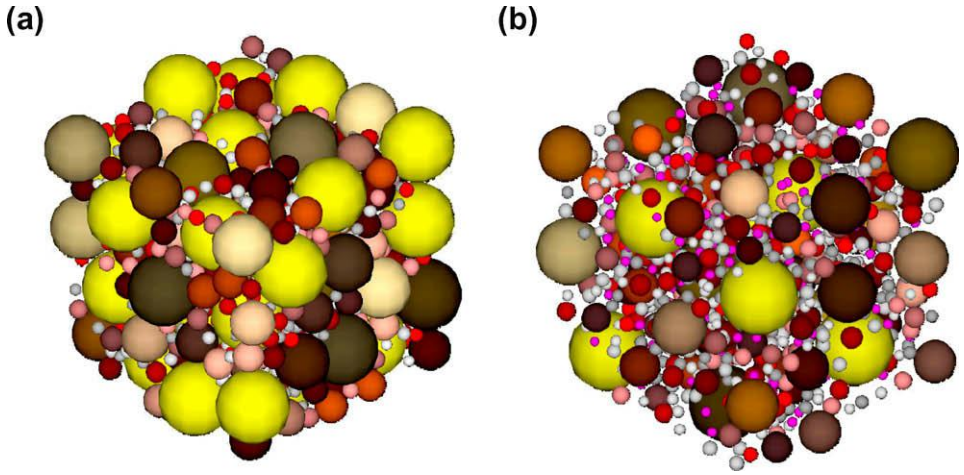

Fig. 3. 3D particle arrangements corresponding to (a) initially pre-packed (IPP) dense packings at $K \leqslant 1$ and (b) continuous distribution (LIP) packings at $K \geqslant 0$ (with particles separation).

and more. In contrast, all SPA–GA experiments resulted in denser packing arrangements with $\eta > 91\%$.

The particle size distribution curves show three characteristic zones (Figs. 2 and 4):

- 1st zone – almost vertical line corresponding to initial pre-packing of spheres of the size close to D_{max}.
- 2nd zone – "structure-forming" range corresponding to spheres in sizes from D_{max} to $(0.5–0.7) * D_{max}$.
- 3rd zone – "void-filling" range corresponding to spheres in sizes less than $(0.5–0.7) * D_{max}$.

It can be observed that the curves, obtained with SPA–GA and representing the densest aggregate assemblies, are "modified" Fuller type or "Initially Pre-Packed" gradings (IPP-gradings) with a predominant volume (45–60%), with the largest particles ranging from D_{max} to $0.7 * D_{max}$ [5,7]. The arrangement of this group of particles results in about 50% of the packing; about 30% of the sphere's volume is represented solely by the spheres of maximal size (D_{max}), which account for 25% of the packing. Finally, a relatively narrow range of particle sizes (from D_{max} to $D_{max}/2$) provides about 60% of the packing (Fig. 2b) [5,7]. This group of particles is considered to be arranged in a manner similar to the "ideal" regular, close-packed lattices (Fig. 3) approaching the condition of the maximum possible value for randomly packed systems (jammed state) [5,7,21]. In this case, the distribution of particles is represented by the relatively narrow range of the sizes. These distributions are characteristic of only "gap-graded" aggregate mixtures that lie outside the limits commonly used in concrete technology (i.e., outside the limits set by Fuller curves with the exponent from 0.45 to 0.7). Similar performance was observed by SPA, but at a low reduction coefficient (−1) and at a wide range of N.

However, in practice, the achievement of well-arranged, initially pre-packed structures is quite difficult using conventional compaction methods due to friction between particles and their irregularity. Therefore, many "real" particulate assemblies could be described by the SPA with high (0 and higher) reduction coefficient. This condition is the case of "Loose Initial Packing" (LIP) arrangement, when the largest particles (from D_{max} to $0.7 * D_{max}$) occupy less than 40% of the volume [5,7]. To achieve high packing degree at a less arranged initial structure (LIP condition), a much wider range of sizes is necessary (Fig. 2a). The graphical representation of this packing arrangement is given in Fig. 3.

The LIP condition can be realized with SPA at a high (0 and higher) reduction coefficient and a wide range of N. The corresponding particle size distribution curve lies between the conventionally employed levels of the Fuller distribution curve with exponent 0.45–0.7 and gap-grading curve of DIN 1045 (Fig. 4).

4.1.2. Packing of aggregates and concrete mix proportioning

As emphasized in the case of high-performance concrete (HPC) [3,7], three fundamental relationships are important for concrete mixture proportioning: w/c – concrete strength; rheological behavior (viscosity or shear stress) of cement paste – slump of concrete mixture; and the optimal aggregate proportioning – workability of concrete. When quantified with corresponding models, these criteria completely define the mixture proportioning and properties of HPC. On other hand, comprehensive modeling of concrete workability and rheological behavior is not possible without detailed knowledge of the arrangement of aggregate's particles, packing degree, and characteristics of porosity.

Similar behavior can be also attributed to common concrete [1,4–6]. It is mutually accepted that better packing provided by the optimization of aggregates is an essential step of concrete mixture proportioning. The common tool currently used to accomplish this task is represented by the Fuller curves. Based on Fuller curves, DIN 1045 specifies the area on the particle size distribution field that includes the preferable gradings (Fig. 4). This area is limited by the grading curves A and B; in Fig. 4, the family of curves with different D_{max} are normalized to the same relative origin using the

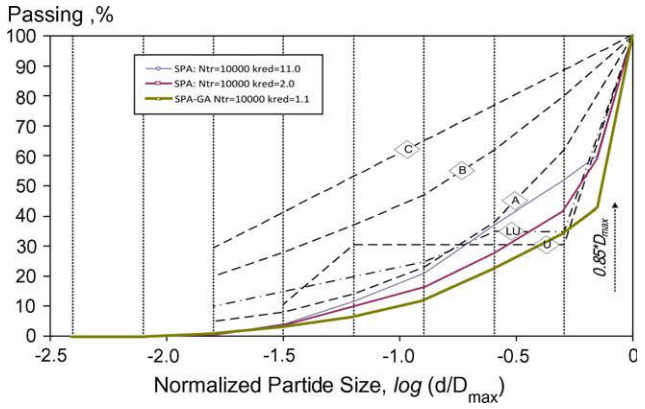

Fig. 4. Particle size distribution curves compared to standard requirements.

formula $\log(d/D_{max})$. The area between the curves B and C is also allowed, but it is normally used for self-consolidating concrete. There is also curve U representing the case of "Gap-Grading" used for zero-slump and roller-compacted concrete. Based on the geometrical calculations, a revised version of "Gap-Grading" (curve LU) was proposed by Kessler [11]. Generally, concrete mixtures with high workability (required for self-compacting and pumped concretes) can be produced with the aggregates adjusted to grading curve B. Conventional vibro-compacted concrete with slump values from 50 to 200 mm can be effectively produced with the aggregates particle size distribution taken as midline between A and B; highly-dense, low-slump concrete is produced preferably with A or U gradings.

The majority of "real" aggregate packings used in concrete technology (i.e., located between the curves A and B) are looser (less arranged) than predicted by the "best packing curves" (these are usually located under the curve A [13,28]). To achieve a higher packing degree, a wider range of particle sizes commonly in concrete technology must be used. The SPA curves acquired with different values of reduction coefficient lie between the curves A and U as specified by DIN 1045. The difference is that the model curves require somewhat larger amounts of coarse aggregate fraction from D_{max} to $0.7 * D_{max}$. Potentially beneficial, such narrow separation of particles is not realized in practice, and real-life grading would use somewhat of a wider size band (from D_{max} to $0.5 * D_{max}$), thereby eliminating this difference. The model curves obtained with SPA–GA are very close to the dense packing LU-curve suggested by Kessler [11].

The packing values (obtained using a vibro-compacting for 30 s according to the procedure adopted from the ASTM C1170-91 (method A) for limestone aggregates corresponding to some of theoretical distributions are compared in Table 3. The "Modified Fuller" curve obtained using SPA at $k_{red} = 11$ ($K = 1$) and $N_{trials} = 10{,}000$ (Fig. 4) demonstrated the best packing degree 75.1% that is very close to the value predicted by the model (75%, Table 2). The A and U gradings resulted with slightly lower packing degree of 74.2% and 74.6%, respectively; and SPA-grading at $k_{red} = 2$ ($K = 0$) and $N_{trials} = 10{,}000$ (Fig. 4) was less dense with 72.4% (which is less than 78% predicted by the model). This difference in last case can be explained by the irregularities of and friction between the aggregate particles, as well as an inability to achieve high compaction gradients, required for better packing.

5. Concluding remarks

1. Implementation of GA module for Sequential Packing Algorithm (SPA) significantly improved the speed of sphere processing and overall enhancement of the computational algorithm. The developed algorithm allows the modeling of real particulate systems used in concrete technology composed of one million particles that differ in the size of up to 150 times. The model's particle size distribution curves acquired with SPA and SPA–GA lie within the limits A and U set by the standards. The developed algorithm and obtained patterns of particulate systems can be used to visualize and model the "real-life" arrangements of concrete aggregates, as well as the behavior of concrete in fresh and hardened state.
2. For a given number of particles, dense packing is achieved either when dense pre-packing is realized or when a wide range of particle sizes is available. The best packing curves with 90% and higher packing degree are presented by the "modified Fuller type", gap or "Initially Pre-Packed" gradings [5]. The characteristic feature of the IPP gradings is related to high (60%) values of packing degree obtained by the narrow range of particle sizes from D_{max} to $D_{max}/2$. The IPP condition is realized with SPA–GA at a wide range of investigated parameters and SPA at low values of reduction coefficient.
3. The opposite case is represented characterized by a "Loose Initial Packing" or LIP condition when a high (80–85%) packing degree is obtained due to the use of a wide range of particle sizes. This condition represented by the SPA models with a reduction coefficient of 0 and higher and at a wide range of N, is characteristic of many "real" particulate assemblies used in concrete.
4. It can be concluded that the best and the densest aggregate packings are not achieved in real systems comprising portland cement and asphalt concrete. The lowest grading curve A corresponding to DIN 1045 and Fuller (at $n = 0.7$) is very close to "Loose Initial Packing" (LIP) condition (obtained at high $K \geqslant 0$ and low $N_{trials} = 10$ k). The main difference between the distributions obtained with models and specified by the standard is that the model suggests using at least 30% of particles with size equal to D_{max}. This is quite difficult to achieve in concrete technology. Still the range of $D_{max} \ldots 3/4\ D_{max}$ can be proposed as critical in providing high packing. According to the modeling results the amount of this aggregate fraction must be at least 40% to achieve optimum (i.e., the densest) packing. The experimental verification of the model demonstrated the applicability of the proposed approach to represent "real-life" concrete

Table 3
Experimental verification of the model gradings.

Curve designation	Theoretical packing degree[a], %	Experimental packing degree, %	
		Rod-tamped	Vibro-compacted ASTM C1170-91 method A
A	–	69.6	74.2
U	–	70.9	74.6
SPA[a]: $k_{red} = 11$ ($K = 1$)	74.88	71.5	75.1
SPA[a]: $k_{red} = 2$ ($K = 0$)	78.47	69.2	72.4

[a] Corresponding to SPA with $N_{trials} = 10{,}000$ as per as Fig. 4.

aggregates. The "Modified Fuller" obtained using SPA at $k_{red} = 11$ ($K = 1$) and $N_{trials} = 10{,}000$ provided the best packing degree 75.1% that is very close to value obtained by the numerical simulation (75%).

5. The developed model can represent virtually any dense particle size distribution used in concrete technology. The developed algorithm, when built into a recursive procedure, can be applied to solve the "classical" packing problem involving the search for the degree of packing for given number of particles and particle size distribution.

Acknowledgements

The financial support provided by CONACYT, PROMEP and PAICYT (Mexico) is acknowledged. Authors gratefully acknowledge the technical support of the Center of Innovation and Information Technologies at Near East University (N. Cyprus).

References

[1] Neville AM. Properties of concrete. Prentice Hall; 2000. 844 pp.
[2] Goltermann P, Johansen V, Palbol L. Packing of aggregates: an alternative tool to determine the optimal aggregate mix. ACI Mater J 1997;94(5):435–43.
[3] de Larrard F, Sedran T. Optimization of ultra high-performance concrete by the use of a packing model. Cem Concr Res 1994;24(6):997–1009.
[4] Vorobiev VA. Application of physical and mathematical methods in concrete research. Visshaya Shkola, Moscow; 1977 [in Russian].
[5] Fuller WB, Thompson SE. The laws of proportioning concrete. ASCE J Transport 1907;59.
[6] Stroeven P, Stroeven M. Assessment of packing characteristics by computer simulation. Cem Concr Res 1999;29:1201–6.
[7] Sobolev K. The Development of a new method for the proportioning of high-performance concrete mixtures. Cem Concr Compos 2004;26(7):901–7.
[8] Garboczi EJ. Three-dimensional mathematical analysis of particle shape using X-ray tomography and spherical harmonics: application to aggregates used in concrete. Cem Concr Res 2002;32(10):1621–38.
[9] Oger L. Étude des Corrélations Structure- Propriétés des Mélanges Granulaires (Study of correlations between structure and properties of granular mixtures). Thèse d'État, Université de Rennes; 1987 [in French].
[10] Andersen PJ, Johansen V. Particle packing and concrete properties. In: Skalny J, Mindess S, editors. Materials science of concrete II. Westerville (OH): The American Ceramic Society; 1995. p. 111–46.
[11] Kessler H-G. Spheres model for gap grading of dense concretes. BFT 1994;11:73–5.
[12] Kwan AKH, Mora CF. Effect of various shape parameters on packing of aggregate particles. Mag Concr Res 2001;53(2):91–100.
[13] Sobolev K, Amirjanov A. The development of a simulation model of the dense packing of large particulate assemblies. Powder Technol 2004;141(1–2):155–60.
[14] Torquato S, Truskett TM, Debenedetti PG. Is random close packing of spheres well defined? Phys Rev Lett 2000;84:2064–7.
[15] Anishchik SV, Medvedev NN. Three-dimensional apollonian packing as a model for dense granular systems. Phys Rev Lett 1995;75:4314–7.
[16] Visscher WM, Bolsterli M. Random packing of equal and unequal spheres in two and three dimensions. Nature 1972;239:504–7.
[17] Scott GD, Kovacs GJ. The radial distributions of a binary mixture of spheres. J Phys D: Appl Phys 1973;6:1007–10.
[18] Coelho D, Thovert J-F, Adler PM. Geometrical and transport properties of random packings of spheres and aspherical particles. Phys Rev E 1997;55:1959–78.
[19] Aim RB, Goff PL. Effet de Paroi dans les Empilements Désordonnés de Sphères et Application à la Porosité de Mélanges Binaires. Powder Technol 1967;1:281–90.
[20] Amirjanov A, Sobolev K. Optimal proportioning of concrete aggregates using a self-adaptive genetic algorithm. Comput Concr 2005;2(5):411–21.
[21] Toufar W, Klose E, Born M. Berechnung der Packungsdichte von Korngemischen. Aufbereitung-Technik 1977;11:603–8.
[22] Jia X, Williams RA. A packing algorithm for particles of arbitrary shapes. Powder Technol 2001;120(3):175–86.
[23] Jullien R, Pavlovitch A, Meakin P. Random packings of spheres built with sequential models. J Phys A: Math Gen 1992;25:4103–13.
[24] Cundall PA. Computer simulation of dense sphere assemblies. In: Satake M, Jenkins JT, editors. Micromechanics of granular materials; 1988. p. 113–23.
[25] Scoppe W. Computer simulation of random packings of hard spheres. Powder Technol 1990;62:189–96.
[26] Yen KZY, Chaki TK. A dynamic simulation of particle rearrangement in powder packings with realistic interactions. J Appl Phys 1992;71:3164–73.
[27] Chang CS, Acheampong KB. Accuracy and stability for static analysis using dynamic formulation in discrete element methods. In: Williams JR, Mustoe GGW, editors. Proceedings of the 2nd international conference on discrete element methods (DEM). IESL Publications; 1993. p. 379–89.
[28] Sobolev K, Amirjanov A. The simulation of particulate materials packing based on the solid suspension model. Adv Powder Technol 2007;18(3):261–71.
[29] Amirjanov A, Sobolev K. Optimization of computer simulation model for packing of concrete aggregates. Particul Sci Technol 2008;26(4):380–95.
[30] Goldberg DE. Genetic algorithms in search, optimization, and machine learning. Reading (MA): Addison-Wesley; 1989.
[31] Coello CAC. Theoretical and numerical constraint-handling techniques used with evolutionary algorithms: a survey of the state of the art. Comput Methods Appl Mech Eng 2002;191:1245–87.
[32] Amirjanov A. The development a changing range genetic algorithm. Comput Method Appl Mech Eng 2006;195:2495.
[33] Runarsson TP, Yao Xin. IEEE Trans Evolut Comput 2000;4:284.