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A novel method for solving the fully neutrosophic linear programming problems

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Abstract

The most widely used technique for solving and optimizing a real-life problem is linear programming (LP), due to its simplicity and efficiency. However, in order to handle the impreciseness in the data, the neutrosophic set theory plays a vital role which makes a simulation of the decision-making process of humans by considering all aspects of decision (i.e., agree, not sure and disagree). By keeping the advantages of it, in the present work, we have introduced the neutrosophic LP models where their parameters are represented with a trapezoidal neutrosophic numbers and presented a technique for solving them. The presented approach has been illustrated with some numerical examples and shows their superiority with the state of the art by comparison. Finally, we conclude that proposed approach is simpler, efficient and capable of solving the LP models as compared to other methods.

Keywords Trapezoidal neutrosophic number · Linear programming · Neutrosophic set · Ranking function

1 Introduction

One of the most extremely used OR methods in real-life problems according to empirical surveys is linear programming [1–4]. It is a mathematical programming which contains a linear objective function and a group of linear equalities and inequalities constraints. The petroleum manufacture was the first and most productive application of linear programming. Well-defined data which contain a greater cost of information are required for LP problems. But in real-life problems, the precision of data is overwhelmingly deceitful and this affects optimal solution of LP problems. Probability distributions failed to transact with inaccurate and unclear information. Also fuzzy sets were introduced by Zadeh [5] to handle vague and imprecise information. But also fuzzy set does not represent vague and imprecise information efficiently, because it considers only the truthiness function. After then, Atanassove [6] introduced the concept of intuitionistic fuzzy set to handle vague and imprecise information, by considering both the truth and falsity function. But also intuitionistic fuzzy set does not simulate human decision-making process. Because the proper decision is fundamentally a problem of arranging and explicate facts the concept of neutrosophic set theory was presented by Smarandache, to handle vague, imprecise and inconsistent information [7–10]. Neutrosophic set theory simulates decision-making process of humans, by considering all aspects of decision-making process. Neutrosophic set is a popularization of fuzzy and intuitionistic fuzzy sets; each element of set had a truth, indeterminacy and falsity membership function. So, neutrosophic set can assimilate inaccurate, vague and maladjusted information efficiently and effectively [11, 12]. We now can say that NLP problem is a problem in which at least one coefficient is represented by a neutrosophic number due to vague, inconsistent and uncertain information. The NLP problems are more useful

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than crisp LP problems because decision maker in his/her formulation of the problem is not forced to make a delicate formulation. The use of NLP problems is recommended to avert unrealistic modeling. In this research, it is the first time to present LP problems in a neutrosophic environment with trapezoidal neutrosophic numbers. Two ranking functions are introduced according to the problem type, for converting NLP problem to crisp problem. The proposed model was applied to both maximization and minimization problems.

The remaining part of this research is marshaled as follows: We survey the pertinent fuzzy and intuitionistic FLP problems literature review in Sect. 2. The important concepts of neutrosophic set arithmetic are presented in Sect. 3. The formularization of NLP models is presented in Sect. 4. The proposed method for solving NLP problems is presented in Sect. 5. Numerical examples are disbanded with the suggested method, a comparison of results with different researchers is illustrated and the drawbacks of existing methods are listed in Sect. 6. Finally, conclusions and future trends are clarified in Sect. 7.

2 Literature review

Linear programming problems in the fuzzy environment have classified into two groups which are, symmetric and non-symmetric problems according to Zimmermann [13]. Objectives and constraints weight are equally significant in symmetric FLP problems, but non-symmetric problem weights of objectives and constraints are not equal [14]. Another classification of FLP problems was introduced by Leung [15]: (1) problems with crisp values of objective and fuzzy values of constraints; (2) problems with crisp values of constraints and fuzzy values of objectives; (3) problems with fuzzy objectives and fuzzy constraints; and finally (4) robust programming problems. Three types of fuzzy linear programming models were proposed by Luhandjula [16], which are flexible, mathematical and fuzzy stochastic programming models. Another six models of FLP problems was introduced by Lnuiguchi et al. [17], which are as follows: flexible, possibility programming, possibility LP by using fuzzy max, possibility linear programming with fuzzy preference relations, possibility linear programming with fuzzy objectives and robust programming. An FLP problem with equality and inequality constraints are introduced by Kumar et al. [18]. Various approaches for disbanding FLP with inequality constraints were proposed by several authors [19–21], by firstly converting FLP problems to its equivalent crisp model and then get the optimal fuzzy solution of the original case. A large number of authors have deliberated different properties of FLP problems and suggested various models for finding solutions. The first introduction of fuzzy programming theory was suggested by Tanaka et al. [22]. The first formulation and solving of FLP problems are presented by Zimmerman [23]. Tanaka and Asai [24] suggested an approach for getting the fuzzy optimal solution of FLP problems. Verdegay solved FLP problems by depending on fuzzification principle of objective [25]. The fuzzified version of mathematical problems was examined by Herrera et al. [26]. An FLP problem with fuzzy values of objective function coefficients were proposed by Zhang et al. [27]. They converted FLP problems into multi-objective problems. Another model of FLP problems with fuzzy values of objective function coefficients and constraints was introduced by Stanciulescu et al. [28]. An FLP model with symmetric trapezoidal fuzzy numbers was presented by Ganesan and Veeramani [29]. They obtained the optimal solution of a problem without converting it to the crisp form. A revised version of Ganesan and Veeramani method was proposed by Ebrahimnejad [30]. A ranking function for arranging trapezoidal fuzzy numbers of FLP problems was introduced by Mahdavi and Naasseri [31]. The idealistic stipulation for FLP problems was derived by Wu [32], by presenting the concept of a non-dominated solution of multi-objective programming. By utilizing a defuzzification function, Wu [33] converted the problem into optimization problems. The full FLP problems were introduced by Lotfi et al. [34]. Some researchers have proposed a ranking function for converting FLP problems into its tantamount crisp LP model and then solving it by standard methods. The primal simplex method was extended by Maleki et al. [35], for solving FLP problems. Tavana and Ebrahimnejad introduced a new approach for solving FLP problems with symmetric trapezoidal fuzzy numbers [36]. The fully intuitionistic FLP problems introduced by Bharati and Singh [37] depend on sign distance between triangular intuitionistic fuzzy numbers. A ranking function was used by Sidhu and Kumar [38] for solving intuitionistic FLP problems. Nagoorgani and Ponnalagu [39] introduced an accuracy function to defuzzify triangular intuitionistic fuzzy number. The previous researches motivated us to propose a study for solving NLP problems. There does not exist any researches which solve neutrosophic linear programming problems with trapezoidal neutrosophic numbers [40–45].

3 Preliminaries

A review of important concepts and definitions of neutrosophic set is presented in this section.

Definition 1 [43] A single-valued neutrosophic set N through X taking the form N={□x, T_N(x), $I_N(x)$, $F_N(x)$□:

x □ X}, where X be a universe of discourse, $T_N(x)$: X→[0, 1], $I_N(x)$: X→[0, 1] and $F_N(x)$: X→[0, 1] with $0 \le T_N(x) + I_N(x) + F_N(x) \le 3$ for all x □ X. $T_N(x)$, $I_N(x)$ and $F_N(x)$ represent truth membership, indeterminacy membership and falsity membership degrees of x to N.

Definition 2 [43] The trapezoidal neutrosophic number $\tilde{A}$ is a neutrosophic set in R with the following truth, indeterminacy and falsity membership functions:

$$T_{\tilde{A}}(x) = \begin{cases} \alpha_{\tilde{A}}\left(\dfrac{x-a_1}{a_2-a_1}\right) & (a_1 \le x \le a_2) \\ \alpha_{\tilde{A}} & (a_2 \le x \le a_3), \\ \alpha_{\tilde{A}} & (a_2 \le x \le a_3) \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{A}}(x) = \begin{cases} \dfrac{(a_2 - x + \theta_{\tilde{A}}(x - a'_1))}{(a_2 - a'_1)} & (a'_1 \le x \le a_2) \\ \theta_{\tilde{A}} & (a_2 \le x \le a_3) \\ \dfrac{(x - a_3 + \theta_{\tilde{A}}(a'_4 - x))}{(a'_4 - a_3)} & (a_3 < x \le a'_4) \\ 1 & \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{A}}(x) = \begin{cases} \dfrac{(a_2 - x + \beta_{\tilde{A}}(x - a''_1))}{(a_2 - a''_1)} & (a''_1 \le x \le a_2) \\ \beta_{\tilde{A}} & (a_2 \le x \le a_3) \\ \dfrac{(x - a_3 + \beta_{\tilde{A}}(a''_4 - x))}{(a''_4 - a_3)} & (a_3 < x \le a''_4) \\ 1 & \text{otherwise} \end{cases} \quad (3)$$

where $\alpha_{\tilde{A}}$, $\theta_{\tilde{A}}$ and $\beta_{\tilde{A}}$ represent the maximum degree of truthiness, minimum degree of indeterminacy, minimum degree of falsity, respectively, $\alpha_{\tilde{A}}$, $\theta_{\tilde{A}}$ and $\beta_{\tilde{A}} \in [0, 1]$.

Also $a''_1 \le a_1 \le a'_1 \le a_2 \le a_3 \le a'_4 \le a_4 \le a''_4$.

The membership functions of trapezoidal neutrosophic number are presented in Fig. 1.

Definition 3 [43] The mathematical operations on two trapezoidal neutrosophic numbers $\tilde{A} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} \rangle$ and $\tilde{B} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{B}}, \theta_{\tilde{B}}, \beta_{\tilde{B}} \rangle$ are as follows:

$$\tilde{A} + \tilde{B} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{B}}, \theta_{\tilde{A}} \vee \theta_{\tilde{B}}, \beta_{\tilde{A}} \vee \beta_{\tilde{B}} \rangle$$

$$\tilde{A} - \tilde{B} = \langle (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{B}}, \theta_{\tilde{A}} \vee \theta_{\tilde{B}}, \beta_{\tilde{A}} \vee \beta_{\tilde{B}} \rangle$$

$$\tilde{A}^{-1} = \left\langle \left(\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right); \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} \right\rangle, \quad \text{where } (\tilde{A} \ne 0)$$

$$\gamma\tilde{A} = \begin{cases} \langle (\gamma a_1, \gamma a_2, \gamma a_3, \gamma a_4); \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} \rangle & \text{if } (\gamma > 0) \\ \langle (\gamma a_4, \gamma a_3, \gamma a_2, \gamma a_1); \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} \rangle & \text{if } (\gamma < 0) \end{cases}$$

$$\frac{\tilde{A}}{\tilde{B}} = \begin{cases} \left\langle \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{B}}, \theta_{\tilde{A}} \vee \theta_{\tilde{B}}, \beta_{\tilde{A}} \vee \beta_{\tilde{B}} \right\rangle & \text{if } (a_4 > 0, b_4 > 0) \\ \left\langle \left(\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{B}}, \theta_{\tilde{A}} \vee \theta_{\tilde{B}}, \beta_{\tilde{A}} \vee \beta_{\tilde{B}} \right\rangle & \text{if } (a_4 \langle 0, b_4 \rangle 0) \\ \left\langle \left(\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}\right); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{B}}, \theta_{\tilde{A}} \vee \theta_{\tilde{B}}, \beta_{\tilde{A}} \vee \beta_{\tilde{B}} \right\rangle & \text{if } (a_4 < 0, b_4 < 0) \end{cases}$$

$$\tilde{A}\tilde{B} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{B}}, \theta_{\tilde{A}} \vee \theta_{\tilde{B}}, \beta_{\tilde{A}} \vee \beta_{\tilde{B}} \rangle & \text{if } (a_4 > 0, b_4 > 0) \\ \langle (a_1 b_4, a_2 b_3, a_3 b_2, a_4 b_1); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{B}}, \theta_{\tilde{A}} \vee \theta_{\tilde{B}}, \beta_{\tilde{A}} \vee \beta_{\tilde{B}} \rangle & \text{if } (a_4 \langle 0, b_4 \rangle 0) \\ \langle (a_4 b_4, a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{B}}, \theta_{\tilde{A}} \vee \theta_{\tilde{B}}, \beta_{\tilde{A}} \vee \beta_{\tilde{B}} \rangle & \text{if } (a_4 < 0, b_4 < 0) \end{cases}$$

Definition 4 A ranking function of neutrosophic numbers is a function R: $N(R) \to R$, where $N(R)$ is a set of neutrosophic numbers defined on set of real numbers, which convert each neutrosophic number into the real line.

Let $\tilde{A} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} \rangle$ and $\tilde{B} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{B}}, \theta_{\tilde{B}}, \beta_{\tilde{B}} \rangle$ are two trapezoidal neutrosophic numbers, then

1. If $R(\tilde{A}) > R(\tilde{B})$ then $\tilde{A} > \tilde{B}$,
2. If $R(\tilde{A}) < R(\tilde{B})$ then $\tilde{A} < \tilde{B}$,
3. If $R(\tilde{A}) = R(\tilde{B})$ then $\tilde{A} = \tilde{B}$.

4 Neutrosophic linear programming problem (NLP)

In this section, various types of NLP problems are presented.

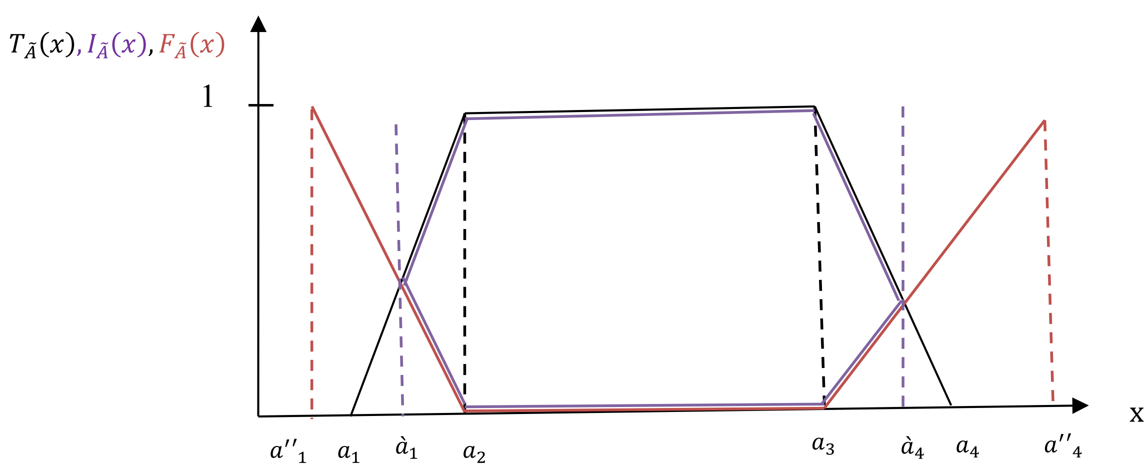

Fig. 1 Truth membership, indeterminacy and falsity membership functions of trapezoidal neutrosophic number

The first type of NLP problem is the problem in which coefficients of objective function variables are represented by trapezoidal neutrosophic numbers, but all other parameters are represented by real numbers.

$$\begin{aligned}&\text{Maximize/minimize } \tilde{Z} \approx \sum_{j=1}^{n} \tilde{c}_j x_j\\&\text{Subject to}\\&\sum_{j=1}^{n} a_{ij} x_j \leq, =, \geq b_i; \quad i = 1, 2, \ldots, m,\\&\quad j = 1, 2, \ldots, n, \quad x_j \geq 0.\end{aligned} \tag{4}$$

In this type of problem, $\tilde{c}_j$ is a trapezoidal neutrosophic number.

The second type of NLP problem is the problem in which objective function variables and coefficients are exemplified by real values but coefficients of constraints variables and right-hand side are represented by trapezoidal neutrosophic numbers.

$$\begin{aligned}&\text{Maximize/minimize } Z = \sum_{j=1}^{n} c_j x_j\\&\text{Subject to}\\&\sum_{j=1}^{n} \tilde{a}_{ij} x_j \tilde{\leq}, \approx, \tilde{\geq} \tilde{b}_i; \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n, \quad x_j \geq 0.\end{aligned} \tag{5}$$

Here, both $\tilde{a}_{ij}$ and $\tilde{b}_i$ are trapezoidal neutrosophic numbers.

The third type of NLP problem is the problem in which all parameters are represented by trapezoidal neutrosophic numbers, except variables are exemplified only by real values.

$$\begin{aligned}&\text{Maximize / minimize } \tilde{Z} \approx \sum_{j=1}^{n} \tilde{c}_j x_j\\&\text{Subject to}\\&\sum_{j=1}^{n} \tilde{a}_{ij} x_j \tilde{\leq}, \approx, \tilde{\geq} \tilde{b}_i; \quad i = 1, 2, \ldots, m,\\&\quad j = 1, 2, \ldots, n, \quad x_j \geq 0.\end{aligned} \tag{6}$$

Here, $\tilde{c}_j, \tilde{a}_{ij}$ and $\tilde{b}_i$ are trapezoidal neutrosophic numbers.

The NLP problem may also be a problem with neutrosophic values for variables, coefficients in goal function and right-hand side of constraints.

$$\begin{aligned}&\text{Maximize/minimize } \tilde{Z} \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j\\&\text{Subject to}\\&\sum_{j=1}^{n} a_{ij} \tilde{x}_j \tilde{\leq}, \approx, \tilde{\geq} \tilde{b}_i; \quad i = 1, 2, \ldots, m,\\&\quad j = 1, 2, \ldots, n, \quad x_j \geq 0.\end{aligned} \tag{7}$$

Here, $\tilde{c}_j, \tilde{x}_j$ and $\tilde{b}_i$ are trapezoidal neutrosophic numbers. Here, $\tilde{x}_j$ is defined as trapezoidal neutrosophic numbers, if authors want to obtain results in the form of neutrosophic numbers. But in reality, any manager or decision maker want to obtain the crisp optimal solution of problem, through considering vague, imprecise and inconsistent information when defining the problem. So, if we obtain the crisp value of decision variables, this problem can be considered as another formulation of NLP (6).

5 Proposed NLP method

A new approach suggested to find the neutrosophic optimal solution of NLP problems is introduced in this section.

Step 1 Let decision makers insert their NLP problem with trapezoidal neutrosophic numbers. Because we always want to maximize truth degree, minimize indeterminacy and falsity degree of information, and then inform decision makers to apply this concept when entering trapezoidal neutrosophic numbers of NLP model.

Step 2 Regarding to definition 4, we propose a ranking function for trapezoidal neutrosophic numbers.

Step 3 Let ($\tilde{a} = a^l, a^{m1}, a^{m2}, a^u, ; T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}$) be a trapezoidal neutrosophic number, where a^l, a^{m1}, a^{m2}, a^u, are lower bound, first, second median value and upper bound for trapezoidal neutrosophic number, respectively. Also $T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}$ are the truth, indeterminacy and falsity degree of trapezoidal number. If NLP problem is a maximization problem, then:

Ranking function for this trapezoidal neutrosophic number is as follows:

$\text{R}(\tilde{a}) = \left(\frac{a^l + a^u + 2\left(a^{m1} + a^{m2}\right)}{2}\right) + \text{confirmation degree.}$

Mathematically, this function can be written as follows:

$$R(\tilde{a}) = \left(\frac{a^l + a^u + 2(a^{m1} + a^{m2})}{2}\right) + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}) \tag{8}$$

If NLP problem is a minimization problem, then:

Ranking function for this trapezoidal neutrosophic number is as follows:

$\text{R}(\tilde{a}) = \left(\frac{a^l + a^u - 3\left(a^{m1} + a^{m2}\right)}{2}\right) + \text{confirmation degree.}$

Mathematically, this function can be written as follows:

$$R(\tilde{a}) = \left(\frac{a^l + a^u - 3(a^{m1} + a^{m2})}{2}\right) + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}). \quad (9)$$

Step 4 According to the type of NLP problem, apply the suitable ranking function to convert each trapezoidal neutrosophic number to its equivalent crisp value. This lead to convert NLP problem to its crisp model.

Step 5 Solve the crisp model using the standard method and obtain the optimal solution of problem.

6 Numerical examples

In this section, to prove the applicability and advantages of our proposed model of NLP problems, we solved the same problem which introduced by Ganesan and Veeramani [29] and Ebrahimnejad and Tavana [36].

The difference between fuzzy set and neutrosophic set is that fuzzy set takes into consideration the truth degree only. But neutrosophic set takes into consideration the truth, indeterminacy and falsity degree. The decision makers and problem solver always seek to maximize the truth degree, minimize indeterminacy and falsity degree. Then, in the following example, we consider truth degree (T)=1, indeterminacy (I) and falsity (F) degree=0, as follows $(1, 0, 0)$ for each trapezoidal neutrosophic number and this called the confirmation degree of each trapezoidal neutrosophic number. We should also note that, according to Ganesan, Veeramani and Ebrahimnejad, Tavana each trapezoidal number is symmetric with the following form:

$$\tilde{a} = \left(a^l, a^u, \alpha, \alpha\right),$$

where a^l, a^u, α, α represented the lower, upper bound and first, second median value of trapezoidal number, respectively. The median values of trapezoidal numbers according to Ganesan, Veeramani and by Ebrahimnejad, Tavana are with equal vales (α). Now let us apply our proposed method on the same problem.

6.1 Example 1

$$\begin{aligned}
&\text{Maximize } \tilde{Z} \approx (13, 15, 2, 2)x_1 + (12, 14, 3, 3)x_2 + (15, 17, 2, 2)x_3 \\
&\text{Subject to} \\
&12x_1 + 13x_2 + 12x_3 \tilde{\leq} (475, 505, 6, 6), \\
&14x_1 + 13x_3 \tilde{\leq} (460, 480, 8, 8), \\
&12x_1 + 15x_2 \tilde{\leq} (465, 495, 5, 5), \\
&x_1, x_2, x_3 \tilde{\geq} \tilde{0}.
\end{aligned} \quad (10)$$

Because this NLP problem is a maximization problem, then by using Eq. (8) each trapezoidal number will convert to its equivalent crisp number. Remember that confirmation degree of each trapezoidal number is (1, 0, 0) according to decision maker opinion as we illustrated previously at the beginning of example. Then, the crisp model of previous problem will be as follows:

$$\begin{aligned}
&\text{Maximize } Z = 19x_1 + 20x_2 + 21x_3 \\
&\text{Subject to} \\
&12x_1 + 13x_2 + 12x_3 \leq 503, \\
&14x_1 + 13x_3 \leq 487, \\
&12x_1 + 15x_2 \leq 491 \\
&x_1, x_2, x_3 \geq 0.
\end{aligned} \quad (11)$$

We can structure the standard form of previous problem (11) as follows:

$$\begin{aligned}
&\text{Maximize } Z = 19x_1 + 20x_2 + 21x_3 \\
&\text{Subject to} \\
&12x_1 + 13x_2 + 12x_3 + s_4 = 503, \\
&14x_1 + 13x_3 + s_5 = 487, \\
&12x_1 + 15x_2 + s_6 = 491, \\
&x_1, x_2, x_3, s_4, s_5, s_6 \geq 0.
\end{aligned} \quad (12)$$

where s_4, s_5, s_6 are slack variables.

The previous standard form can be solved by the simplex approach. The initial tableau of simplex is presented in Table 1.

The coming variable in Table 2 is x_3 and departing variable is s_5.

The entering variable is x_2 and leaving variable is s_4 as shown in Table 3.

Table 1 Initial simplex form

Basic variables	x_1	x_2	x_3	s_4	s_5	s_6	RHS
s_4	12	13	12	1	0	0	503
s_5	14	0	13	0	1	0	487
s_6	12	15	0	0	0	1	491
Z	19	20	21	0	0	0	0

Table 2 First simplex form

Basic variables	x_1	x_2	x_3	s_4	s_5	s_6	RHS
s_4	− 12/13	13	0	1	− 12/13	0	695/13
x_3	14/13	0	1	0	1/13	0	487/13
s_6	12	15	0	0	0	1	491
Z	− 47/13	20	0	0	− 21/13	0	10,227/13

Table 3 Optimal form

Basic variables	x_1	x_2	x_3	s_4	s_5	s_6	RHS
x_2	− 12/169	1	0	1/13	− 12/169	0	695/169
x_3	14/13	0	1	0	1/13	0	487/13
s_6	2208/169	0	0	− 15/13	180/169	1	429
Z	− 371/169	0	0	− 20/13	− 33/169	0	869

6.2 Comparisons between our proposed model and other existing models

By comparing proposed model results with Ebrahimnejad and Tavana [36] results of the same problem, we noted that:

1. Our proposed model results are better than Ebrahimnejad and Tavana results. Let us look at the optimal tableau of our proposed model as shown in Table 3, it is obvious that the objective function value equal 869 but in Ebrahimnejad and Tavana, the objective function equal 635 by knowing that, the problem is a maximization problem. To make this more obvious, let us introduce the optimal tableau of Ebrahimnejad and Tavana model as presented in Table 4.
2. Ebrahimnejad and Tavana proposed their model to solve only symmetric trapezoidal numbers. But our model can solve symmetric and non-symmetric numbers.
3. When entering symmetric trapezoidal numbers of Ebrahimnejad and Tavana, it take the following form:

$\tilde{a} = \left(a^l, a^u, \alpha, \alpha\right)$, and they did not utilize the value of α in their calculations of ranking function for obtaining the equivalent crisp value, so let us ask ourselves a question "what is the rule of α?". But in our proposed model, we take all values into considerations. Our ranking function has not any missing values of trapezoidal numbers, and then it is very accurate and comprehensive.

4. As we know,a^l, a^u, α, α represented the lower, upper bound, first and second median value of trapezoidal number, respectively. Because two values of α are equals, then the triangular numbers will be more logical than trapezoidal numbers.
5. To solve a problem with not symmetric trapezoidal numbers using Ebrahimnejad and Tavana method, we need to approximate all not symmetric trapezoidal numbers into the closest symmetric numbers. This approximation will make obtained results which are not delicate.
6. The big drawback of Ebrahimnejad and Tavana fuzzy model is the taking of truthiness function only. But in real life, the decision-making process takes the following form "agree, not sure and disagree." We treated this drawback in our model by using neutrosophic. Since, beside the truth function, we take into account the indeterminacy and falsity function.

Table 4 Ebrahimnejad and Tavana optimal tableau

Basis	x_1	x_2	x_3	s_4	s_5	s_6	RHS
x_2	− 12/169	1	0	1/13	− 12/169	0	730/169
x_3	14/13	0	1	0	1/13	0	470/169
s_6	1848/169	0	0	− 15/13	180/169	1	70,170/169
Z	42/13	0	0	1	52/169	0	634.6

Also by comparing our model with Ganesan and Veeramani at the same problem, we also noted that:

1. Our model is more simple and efficient than Ganesan and Veeramani model.
2. Since obtained results of Ebrahimnejad, Tavana and Ganesan and Veeramani are equals then, our results are also better than Ganesan and Veeramani model.
3. Our model represents reality efficiently than Ganesan and Veeramani model, because we consider all aspects of decision-making process in our calculations (i.e., the truthiness, indeterminacy and falsity degree).
4. Ganesan and Veeramani model represented to solve only the symmetric trapezoidal numbers. Our model can solve both the symmetric and non-symmetric.

Also, by comparing our model with Kumar et al. [18] for solving the same problem we founded that:

1. In their model, they convert the FLP problem to its tantamount crisp model. But their model has more variables and constraints.
2. Their models increase the complexity of solving linear programming problem by simplex algorithm.
3. Our model reduces complexity of problem, by reducing the number of constraints and variables.
4. Their model is a time-consuming and complex, but our model is not.
5. Also our model represents reality efficiently and better than their model.

By solving the previous example according to Saati et al. [44] proposed method, then the model will be as follows:

$$\begin{aligned}
&\text{Maximize } Z = 13x_1 + 12x_2 + 15x_3\\
&\text{Subject to}\\
&12x_1^l + 13x_2^l + 12x_3^l \le 475,\\
&12x_1^u + 13x_2^u + 12x_3^u \le 505,\\
&12x_1^{m1} + 13x_2^{m1} + 12x_3^{m1} \le 6,\\
&12x_1^{m2} + 13x_2^{m2} + 12x_3^{m2} \le 6,\\
&14x_1^l + 13x_3^l \le 460,\\
&14x_1^u + 13x_3^u \le 480,\\
&14x_1^{m1} + 13x_3^{m1} \le 8,\\
&14x_1^{m2} + 13x_3^{m2} \le 8,\\
&12x_1^l + 15x_2^l \le 465,\\
&12x_1^u + 15x_2^u \le 495,\\
&12x_1^{m1} + 15x_2^{m1} \le 5,\\
&12x_1^{m2} + 15x_2^{m2} \le 5,\\
&x_1^l + x_1^u \ge 0,\\
&x_2^l + x_2^u \ge 0,\\
&x_3^l + x_3^u \ge 0,\\
&x_1^{m1} + x_1^{m2} \ge 0,\\
&x_2^{m1} + x_2^{m2} \ge 0,\\
&x_3^{m1} + x_3^{m2} \ge 0.
\end{aligned} \tag{13}$$

As an effect, the numbers of constraints and variables are increased, and this lead to increase complexity of problem, increase the space of recording binary bits and also increase computational time when solving it by simplex method. If the numbers of constraints of the original problem are increased, then the solution will become very difficult to apply. But our proposed method solves the same problem with less variables and constraints, and then, with less complexity and also less computational time when solving by simplex method.

6.3 Example 2

In this example, we solve a NLP problem with trapezoidal neutrosophic numbers. The order of element for trapezoidal neutrosophic numbers is as follows: lower, first median value, second median value and finally the upper bound. The decision makers' confirmation degree about each value of trapezoidal neutrosophic number is (0.9, 0.1, 0.1). This example belongs to the second classification of NLP problems as listed in Sect. 4.

Table 5 Initial simplex form

Basic variables	x_1	x_2	s_3	s_4	s_5	RHS
s_3	33	46	1	0	0	90,041
s_4	53	15	0	1	0	48,046
s_5	44	34	0	0	1	56,031
Z	25	48	0	0	0	0

$$\begin{aligned}
&\text{Maximize } Z = 25x_1 + 48x_2\\
&\text{Subject to}\\
&(14, 15, 17, 18)x_1 + (25, 30, 34, 38)x_2\\
&\quad \tilde{\le} (44,980, 45,000, 45,030, 45,070)\\
&(21,\ 24,\ 26,\ 33)x_1 + (4,\ 6,\ 8,\ 11)x_2\\
&\quad \tilde{\le} (23,980, 24,000, 24,050, 24,060)\\
&(17,\ 21,\ 22,\ 26)x_1 + (12, 14, 19, 22)x_2\\
&\quad \tilde{\le} (27,990, 28,000, 28,030, 28,040)\\
&\tilde{x}_1, \tilde{x}_2 \tilde{\ge} \tilde{0}.
\end{aligned} \tag{14}$$

By using Eq. (8), each trapezoidal number will convert to its equivalent crisp number. Then, the crisp model of previous problem will be as follows:

$$\begin{aligned}
&\text{Maximize } Z = 25x_1 + 48x_2\\
&\text{Subject to}\\
&33x_1 + 64x_2 \le 90,041,\\
&53x_1 + 15x_2 \le 48,046,\\
&44x_1 + 34x_2 \le 56,031,\\
&x_1, x_2 \ge 0.
\end{aligned} \tag{15}$$

We can structure the standard form of previous problem (15) as follows:

$$\begin{aligned}
&\text{Maximize } \ Z = 25x_1 + 48x_2\\
&\text{Subject to}\\
&33x_1 + 64x_2 + s_3 = 90,041\\
&53x_1 + 15x_2 + s_4 = 48,046\\
&44x_1 + 34x_2 + s_5 = 56,031\\
&x_1, x_2, s_3, s_4, s_5 \ge 0.
\end{aligned} \tag{16}$$

where s_3, s_4, s_5 are slack variables.

The previous standard form can be solved by the simplex approach. The initial tableau of simplex is presented in Table 5. The entering variable in Table 6 is x_2 and leaving variable is s_3.

The coming variable is x_1 and departing variable is s_5 as in Table 7.

Table 6 First simplex form

Basic variables	x_1	x_2	s_3	s_4	s_5	RHS
x_2	33/64	1	1/64	0	0	1406.89
s_4	2897/64	0	− 15/64	1	0	26,942.6
s_5	847/32	0	− 17/32	0	1	8196.72
Z	0.25	0	− 0.75	0	0	67,530.8

Table 7 Optimal form

Basic variables	x_1	x_2	s_3	s_4	s_5	RHS
x_2	0	1	2/77	0	− 3/154	1247.21
s_4	0	0	571/847	1	− 1.71015	12,925
x_1	1	0	− 17/847	0	32/847	23,845/77
Z	0	0	− 631/847	0	− 8/847	67,608.2

Table 8 Optimal simplex form

Basis	x_1	x_2	s_3	s_4	RHS
s_3	0	− 7/3	1	− 2/3	10
x_1	1	4/3	0	− 1/3	2
Z	0	2	0	2	12

Table 9 Departments

Products	Wiring	Drilling	Assembly	Inspection	Unit profit
P1	0.5	3	2	0.5	$\tilde{9}$$
P2	1.5	1	4	1	$\widetilde{12}$$
P3	1.5	2	1	0.5	$\widetilde{15}$$
P4	1	3	2	0.5	$\widetilde{11}$$

6.4 Example 3

Let us introduce another type of problems in this example and making a comparison with other research at the same example.

By solving the same problem which introduced by Saati et al. [44]:

$$\begin{aligned}&\text{Minimize } Z = 6x_1 + 10x_2\\&\text{Subject to}\\&2x_1 + 5x_2 \tilde{\geq} (5, 8, 3, 13),\\&3x_1 + 4x_2 \tilde{\geq} (6, 0, 4, 16),\\&x_1, x_2 \tilde{\geq} \tilde{0}.\end{aligned} \tag{17}$$

Let confirmation degree is (1, 0, 0) according to our assumptions and note that, here the order of trapezoidal neutrosophic number is as follows: lower bound, first, second median value and finally the upper bound, respectively. Let us use Eq. (9) for transforming the previous model to its crisp model as follows:

$$\begin{aligned}&\text{Minimize } Z = 6x_1 + 10x_2\\&\text{Subject to}\\&2x_1 + 5x_2 \geq -6, \qquad .\\&3x_1 + 4x_2 \geq 6,\\&x_1, x_2 \geq 0.\end{aligned} \tag{18}$$

The previous problem can be solved by the simplex approach. The optimal tableau of simplex method is presented in Table 8.

From the previous table, the value of objective function =12, $x_1 = 2$ and $x_2 = 0$.

When Saati et al. [35] solved the previous example, the results are nearly equal with our result. Since the value of Z according to their model is equal to 12.857, the value of $x_1 = 1.429$ and $x_2 = 0.429$. It is obvious that two approach results are nearly equal, but our proposed method has several advantages over their method:

1. We obtain the results which also obtained by Saati et al. [44] but with easy and simple method.
2. Number of constraints in our model is the same of the original model, but when Saati solved their model, the number of variables and constraints is significantly increased. Since in Saati et al. [44] model, number of constraints of the previous problem becomes 20 constraints when they trying to solve the previous problem.
3. Due to the big increase in number of variables and constraints of Saati model, the complexity of solving the problem by simplex will increase and computational time will increase sure.
4. Their proposed approach is difficult to apply in large scale of problems.
5. Also their approach does not represent vague, inconsistent information efficiently.

6.5 Case study

A company for electronic industries manufactures four technical products for aerospace companies that conclude NASA contracts. The outputs must get through four parts before they are shipped. These departments are: Wiring, Drilling, Assembly and finally Inspection. The required time for each unit manufactured and its profit is presented in Table 9. The minimum production quantity for fulfilling contracts monthly is presented in Table 10. The objective of company is to produce products in such quantities for maximizing the total profits.

Table 10 Time capacity and minimum production level

Departments	Capacity (in hours)	Products	Minimum production level
Wiring	$\widetilde{1500}$	P1	$\widetilde{150}$
Drilling	$\widetilde{2350}$	P2	$\widetilde{100}$
Assembly	$\widetilde{2600}$	P3	$\widetilde{300}$
Inspection	$\widetilde{1200}$	P4	$\widetilde{400}$

The confirmation degree of previous information according to decision makers' opinions is (0.9, 0.1, 0.1).

Let number of units of p1 produced=x_1,

Let number of units of p2 produced=x_2,

Let number of units of p3 produced=x_3,

Let number of units of p4 produced=x_4.

The formulation of previous problem is as follows:

$$\begin{aligned}
&\text{Maximize } \tilde{Z} \approx \tilde{9}x_1 + \widetilde{12}x_2 + \widetilde{15}x_3 + \widetilde{11}x_4 \\
&\text{Subject to} \\
&0.5x_1 + 1.5x_2 + 1.5x_3 + x_4 \leq \widetilde{1500}, \\
&3x_1 + x_2 + 2x_3 + 3x_4 \leq \widetilde{2350}, \\
&2x_1 + 4x_2 + x_3 + 2x_4 \leq \widetilde{2600}, \\
&0.5x_1 + x_2 + 0.5x_3 + 0.5x_4 \leq \widetilde{1200}, \\
&x_1 \geq \widetilde{150}, \\
&x_2 \geq \widetilde{100}, \\
&x_3 \geq \widetilde{300}, \\
&x_4 \geq \widetilde{400}. \\
&x_1, x_2, x_3, x_4 \geq \tilde{0}.
\end{aligned} \tag{19}$$

Note that the values of each neutrosophic number represented by a trapezoidal neutrosophic number as follows:

$$\begin{aligned}
&\tilde{9} = (6, 8, 9, 12), \widetilde{12} = (9, 10, 12, 14), \\
&\widetilde{15} = (12, 13, 15, 17), \widetilde{11} = (8, 9, 11, 13), \\
&\widetilde{150} = (120, 130, 150, 170), \\
&\widetilde{100} = (70, 80, 100, 120), \\
&\widetilde{300} = (270, 280, 300, 320), \widetilde{400} = (370, 380, 400, 420), \\
&\widetilde{1500} = (1200, 1300, 1500, 1700), \\
&\widetilde{2350} = (2200, 2250, 2350, 2400) \\
&\widetilde{2600} = (2200, 2400, 2600, 2800), \\
&\widetilde{1200} = (1000, 1100, 1200, 1300).
\end{aligned}$$

By using Eq. (8), the previous problem transform to the following crisp model as follows:

$$\begin{aligned}
&\text{Maximize } Z = 27x_1 + 34x_2 + 43x_3 + 31x_4 \\
&\text{Subject to} \\
&0.5x_1 + 1.5x_2 + 1.5x_3 + x_4 \leq 4251, \\
&3x_1 + x_2 + 2x_3 + 3x_4 \leq 6901, \\
&2x_1 + 4x_2 + x_3 + 2x_4 \leq 7501, \\
&0.5x_1 + x_2 + 0.5x_3 + 0.5x_4 \leq 3451, \\
&x_1 \geq 426, \\
&x_2 \geq 276, \\
&x_3 \geq 876, \\
&x_4 \geq 1176. \\
&x_1, x_2, x_3, x_4 \geq 0.
\end{aligned} \tag{20}$$

By solving the previous model using simplex approach, the results are as follows:

$$\begin{aligned}
&x_1 = 426, \\
&x_2 = 343, \\
&x_3 = 876, \\
&x_4 = 1176, \\
&Z = 97,288.
\end{aligned}$$

7 Conclusions and research directions

By applying the neutrosophic set concept to the linear programming problems, we treated imprecise, vague and inconsistent information efficiently. We also have a better representation of reality through considering all aspects of the decision-making process. We proposed two ranking functions for converting trapezoidal neutrosophic numbers to its equivalent crisp values. The first ranking function is for maximization problems and the second-ranking function is for minimization problems. After using the suitable ranking function and transforming the problem to its equivalent crisp model, then we solve the problem using the standard methods. By comparing our proposed model with other existing fuzzy models, we concluded that our proposed model is simpler, efficient and achieve better results than other researchers. It is also revealed that proposed method is equivalently applied for solving with the symmetric and non-symmetric trapezoidal numbers.