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Supply chain management with market economics

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Abstract

Supply chain management (SCM) is now recognised as one of the best means by which enterprises can make instant improvements to their business strategies and operations. SCM, however, is generally based on the simple theory of constraints (TOC) concept, and is not always concerned with Pareto optimal solutions in product distribution. Since market price systems constitute a well-understood class of mechanisms that under certain conditions provide effective decentralisation of decision making with minimal communication overhead, we propose SCM based on market-oriented programming in this paper. In market-oriented programming, we take a metaphor of economy computing multi-agent behaviour literally, and directly implement the distributed computation as a market price system. We define the agent activities to negotiate the tradeoffs of acquiring different resources, so as to realise the multi-echelon optimisation. Several simulation experiments on the supply chain model with multi-echelon structure clarify the market dynamics that emerge through the agent negotiations. It is confirmed that careful constructions of the decision process according to economic principles can lead to Pareto optimal resource allocations in SCM, and the behaviour of the system can be analysed in economic terms. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Supply chain management; Market-oriented programming; Product distribution; Multi-agent paradigm; Distribution system

1. Introduction

During the last few years the focus has shifted from factory level to enterprise level due to the increasing global presence of the companies. Supply chain management (SCM) is now recognised as one of the best means by which enterprises can make instant improvements to their business strategies [1]. Manufacturers and suppliers have to decide if they would like to form close relationships not to have partial solutions. Real benefits can only be attained by sincere commitment from each of the partner to use what is proposed. Sharing of information is central to the optimisation of resource allocation (i.e., product distribution) in the supply chain. SCM is generally based on the simple theory of constraints (TOC) with throughput-based costing method, and conducts effective strategies in the enterprise level by DBR (Drum, Buffer and Lope) concept [2].

The management of physical flow of products amongst the nodes of the supply chain comes under the intensive study of effective operation in SCM. Since supply chains consist of several layers of business units, resource allocation is a quite important operational criterion at workshop level in

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SCM. As the number of potential business units in the supply chain increases, an effective management on product distribution (i.e., multi-echelon optimisation) plays a more important role in dynamic environment. Current SCM concept does not deal with the problem, because TOC does not handle combinatorial optimisation problem in the resource allocation.

Recently the use of multi-agent system in large-sized complex system is increasing [3]. The multi-agent paradigm has several characteristics, such as autonomy, pro-activeness, social ability, and emergence. In this paradigm, a global goal of the whole system is achieved as the aggregation of their local objectives with their negotiation. In supply chain networks, each business unit behaves independently and autonomously with simple goals of achieving local optimum. The situation is quite similar to the distributed decision making mechanism in multi-agent paradigm, and it is natural to model supply chain networks through multi-agent programming. In such an environment, each agent represents the independent business unit with conflicting and competing individual requirements, and may possess localised information relevant to their utilities. To recognise this independence, we treat the business units as agents, allowing each of them to decide autonomously how to deploy resources under their control in service of their interests.

Within this model, a distributed SCM can be analysed according to the following properties:

- Self-interest agents can make effective decisions with local information, without knowing the private information and strategies of other agents.
- The method requires minimal communication overhead.
- Solutions do not waste resources. If there is some way to make some agent better off without harming others, it should be done. A solution that cannot be improved in this way is called Pareto optimal.

Assuming that a resource allocation problem in SCM must be decentralised in considering a practical application, market concept can provide several advantages:

(i) Markets are naturally distributed and agents make their own decisions about how to bid based on the prices and their own utilities of the goods.
(ii) Communication is limited to the exchange of bids and process between agents and the market mechanism. In particular settings, it can be shown that price systems minimise the dimensionality of messages required to determine Pareto optimal allocation.
(iii) Since agent must back their representations with exchange offers, some mechanism can elicit the information necessary to achieve Pareto and system optima in some well-categorised situations.

Market-oriented programming is a multi-agent-based concept to facilitate distributed problem solving. In the market-oriented programming, we take the metaphor of an economy computing multi-agent behaviour literally, and directly implement the distributed computation as a market price system. In the market-oriented programming approach to distributed problem solving, the resource allocation for a set of computational agents is derived by computing competitive market of an artificial economy [4–6].

In this paper, we formulate supply chain model as a discrete resource allocation problem with supply/demand agents, and demonstrate the applicability of economic analysis to this framework by simulation experiments. Finally, we prove that the market mechanism can provide several advantages on resource allocation in SCM. Needless to say, the term 'resource allocation' in this paper corresponds to 'product distribution' at workshop level in practical SCM.

2. Market-oriented programming

2.1. Market-based approach

In economics, the concept of a set of interrelated goods in balance is called general equilibrium. The general equilibrium theory guarantees a Pareto optimal solution at competitive equilibrium in perfect competitive market [7]. The connection between computation and general equilibrium is not all foreign to economists, who often appeal to the metaphor of market systems computing the activities of the agents involved [8].

The theory of general equilibrium provides the foundation for a general approach to the construction of distributed planning system based on price mechanism. In this approach, the constituent planning agents are regarded as suppliers and demanders in an artificial economy. Their individual activities are defined in terms of production and consumption of resources. Interactions amongst agents are cast as exchanges, the terms of which are mediated by the underlying economic mechanism, or protocol.

2.2. Bidding mechanism in market-oriented programming

Market-oriented programming is the general approach of deriving solutions to distributed resource allocation problems by computing the competitive equilibrium of an artificial economy [4,5]. It involves an iterative adjustment of prices based on the reactions of the agent in the market. Bidding mechanism in market-oriented programming is shown in Fig. 1.

Let $P_t(s)$ be the price of resource s at time t. $f_t ms$ and $g_t ns$ represent the supply function [7] (defined in 3.5) of supplier m on resource s at time t and the demand function [7] (defined in 3.5) of demander n on resource s at time t, respectively.

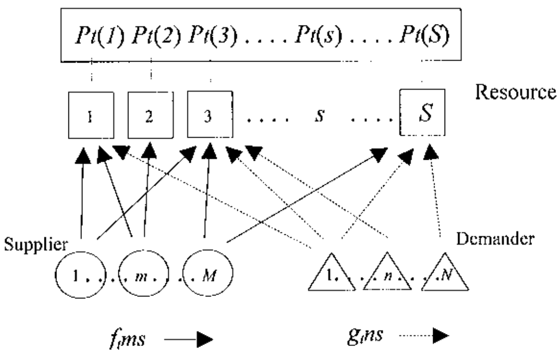

Fig. 1. Bidding mechanism.

The bidding mechanism computes an equilibrium price in each separate market. It involves an iterative adjustment of prices based on reactions of agents in the market. Agent s submits supply and demand functions ($f_t ms$ and $g_t ns$) and the auction adjusts individual prices to clear, rather than adjusting the entire price vector by some increment. The mechanism associates an auction with each distinct resource. Agents act in the market by submitting bids to auctions. In this paper bids specify a correspondence between prices and quantities of the resource that the agent offers to demand or supply as a basic study. Given bids from all interested agents, the auction derives a market-clearing price.

Each agent maintains an agenda of bid tasks, specifying in which it must update its bid or compute a new one. The bidding process is highly distributed, in that each agent need communicate directly only with the auctions for the resources of interest. Each of these interaction concerns only a single resource; the auctions never coordinate with each other. Agents need not negotiate directly with other agents, nor even know of each other's existence.

As new bids are received at the auctions, the previously computed clearing price becomes obsolete. Periodically, each auction computes a new clearing price if any new or updated bids have been received, and posts it on the tote board. When a price is updated, this may invalidate some of an agent's outstanding bids, since these were computed under the assumption that price for remaining resources were fixed at previous value. On finding out about a price change, an agent arguments its task agenda to include the potentially affected bids.

At all times, the market-oriented mechanism maintains a vector of going prices and quantities that would be exchanged at those prices. While the agents have nonempty bid agendas or the auctions new bids, some or all resources may be in disequilibrium. When all auctions clear and all agendas are exhausted, however, the economy is in competitive equilibrium.

2.3. Market-oriented programming and SCM

Agent activities in terms of products required and supplied are defined so as to reduce an agent's decision problem to evaluate the tradeoffs of acquiring different products in market-oriented programming. These tradeoffs are represented in market prices, which define common scale of value across the various products. The problem for designers of computational markets is to specify the strategy by which agent interactions determine prices [4].

Obviously supply chain model is well structured for market-oriented programming, and that means the proposed concept takes advantage of the theory. A Pareto optimal solution, which is conducted by microeconomics, is attainable in resource allocation problem in SCM.

In the next section, we define several functions that formulate agent's strategy for the resource allocation problem in SCM. Budget constraint of each agent is also considered in our definitions for practical use.

3. Agent definition

3.1. Preliminaries

Several variables to formulate agent utilities in this paper are defined as follows:

X_{ki}	Input of resource i in agent k
y_{kj}	Yield of resource j in agent k
p_I	Purchase price of resource i per unit
P_i	Sales price of resource i per unit
E_k	Profit function of agent k
C_k	Cost of agent k
S_k	Sales of agent k
${}_{\max}C_k$	Budget of agent k
$l(i,j)$	$(=l)$: Index of production function from resource i to resource j
F_{kl}	lth production function in agent k
X_{kl}	Input resource amount into production function f_{kl}
Y_{kl}	Output resource amount from production function f_{kl}
E_{kl}	Profit by production function f_{kl}
c_{kl}	Cost by production function f_{kl}
s_{kl}	Sales by production function f_{kl}

3.2. Production function

Suppose supply agent k has a production function f_k described in Eq. (1).

$$Y_k = f_k(X_k), \tag{1}$$

where X_k and Y_k denote a set of input resources and yield resources in agent k described in Eqs. (2) and (3), respectively:

$$X_k = \{x_{k1}, \ldots, x_{km}\}, \tag{2}$$

$$Y_k = \{y_{k1}, \ldots, y_{kn}\}. \tag{3}$$

In this paper, we adopt Cobb–Douglas function [7] as a production function described in Eq. (4). Since Cobb–Douglas function handles economical scale in the market by index constant b, and in $0 < b < 1$ the production function is defined as a concave down function, in other words, a diminishing returns function. If production function is defined as concave down, market prices are established at a predictable level in the general equilibrium theory.

$$y = ax^b \quad (\text{where } 0 < a, 0 < b < 1). \tag{4}$$

Then the production function f_{kl} of agent k for input–output resource set $l(i,j) = l$ is given by

$$y_{kl} = f_{kl}(x_{kl}) = a_{kl}x_{kl}^{b_{kl}}, \tag{5}$$

where x_{kl} and y_{kl} denote the amount of input resource i for f_{kl} and the amount of yield resource j for f_{kl}, respectively. Then x_{ki} and y_{ki} are defined as

$$x_{ki} = \sum_j x_{kl(i,j)}, \tag{6}$$

$$y_{kj} = \sum_i y_{kl(i,j)}. \tag{7}$$

3.3. Profit function

Suppose a set of single unit purchase prices for a resource set $\{x_{k1}, \ldots, x_{km}\}$ is $\{p_1, \ldots, p_m\}$, and a set of single unit sales prices for a resource set

$\{y_{k1}, \ldots, y_{kn}\}$ is $\{P_1, \ldots, P_n\}$, then the total production cost C_k of agent k is defined as

$$c_{kl(i,j)} = p_i x_{kl}, \tag{8}$$

$$C_k = \sum_l c_{kl}, \tag{9}$$

and the total sales S_k of agent k is defined as

$$s_{kl(i,j)} = P_j y_{kl}, \tag{10}$$

$$S_k = \sum_l s_{kl}. \tag{11}$$

Then the profit function E_k of agent k is finally acquired as

$$E_{kl} = s_{kl} - c_{kl}, \tag{12}$$

$$E_k = \sum_l E_{kl}. \tag{13}$$

3.4. *Profit maximise theorem under budget constraint*

In this paper, budget constraint of each agent is considered so as to realise our market model. Suppose the maximum budget of agent k is ${}_{\max}C_k$, then we have

$$C_k = \sum_l c_{kl} \leqslant {}_{\max}C_k \tag{14}$$

and agent k should behave to maximise its profit E_k autonomously.

The basic principle of agents is to maximise their profits under the budget constraints. Their activities should follow the newly proposed theorem, named Profit Maximise Theorem, shown below.

Theorem. *Profit function E_k of agent k is maximised by minimised r_k, which satisfies the following conditions*:

$$\forall l: \quad \frac{\partial E_k}{\partial c_{kl}} = r_k \; (r_k \geqslant 0) \cap C_k \leqslant {}_{\max}C_k \tag{15}$$

subject to

$$f_k \text{ is differentiable in any } x \in X_k, \quad \forall l: \quad \left.\frac{\partial f_{kl}}{\partial x_{kl}}\right|_{x_{kl}=x} > \left.\frac{\partial f_{kl}}{\partial x_{kl}}\right|_{x_{kl}=x+\Delta}. \tag{16}$$

We have the following Eq. (17) by Eqs. (8), (10), (12), (13):

$$\begin{aligned} \frac{\partial E_k}{\partial c_{kl(i,j)}} &= \frac{\partial E_{kl}}{\partial c_{kl}} = \frac{\partial}{\partial c_{kl}}(s_{kl} - c_{kl}) \\ &= \frac{\partial}{\partial c_{kl}}[P_j f_{kl}(c_{kl}/p_i) - c_{kl}] \\ &= \frac{P_j}{p_i} = f'_{kl}(c_{kl}/p_i) - 1. \end{aligned} \tag{17}$$

The proof of the theorem is given in Appendix A.

3.5. *Demand/supply function definitions*

Since Cobb–Douglas function shown in (4) is differentiable and

$$\frac{\partial f_{kl}}{\partial x_{kl}} = a_{kl} b_{kl} x^{b_{kl}-1} > a_{kl} b_{kl} (x+\Delta)^{b_{kl}-1} > 0, \tag{18}$$

then the proposed product function (5) perfectly satisfies the conditions (16). Demand function x_{kl}, which maximises the agent's profit, is obtained by the Profit Maximise Theorem as follows:

$$\begin{aligned} \frac{\partial E_k}{\partial c_{kl(i,j)}} &= \frac{\partial E_{kl}}{\partial c_{kl}} = \frac{\partial}{\partial c_{kl}}[P_j a_{kl} (c_{kl}/p_i)^{b_{kl}} - c_{kl}] \\ &= a_{kl} b_{kl} P_j c_{kl}^{b_{kl}-1} p_i^{-b_{kl}} - 1 \\ &= a_{kl} b_{kl} P_j p_i^{-1} x_{kl}^{b_{kl}-1} - 1 = r_k. \end{aligned} \tag{19}$$

Then, we have

$$x_{kl(i,j)} = [p_i(r_k+1)/a_{kl} b_{kl} P_j]^{b_{kl}/b_{kl}-1}. \tag{20}$$

Supply function y_{kl}, which maximises the profit, is also obtained by Eqs. (5), (20) as follows:

$$y_{kl(i,j)} = a_{kl}[p_i(r_k+1)/a_{kl} b_{kl} P_j]^{b_{kl}/b_{kl}-1}. \tag{21}$$

We denote a concrete meaning of the Profit Maximise Theorem. It is obvious to maximise the production function f_{kl} at $r_k = 0$ by (19), because the function is defined as concave down type. Then agent k has the maximum profit at $r_k = 0$, if it satisfies the budget constraint ($C_k \leqslant {}_{\max}C_k$). However, if agent k breaks the budget constraint ($C_k > {}_{\max}C_k$), then it has to reduce some amount of input resource x_{kl} to satisfy the constraint. The theorem conducts it should adjust the amount of input resources to have the equivalent value of $\partial E_{kl}/\partial c_{kl}$ in all the production function f_{kl}. If the value r_k increases, the amount of the demanded resources decreases, and that leads to reduce the cost C_k. Then the minimised r_k in $C_k \leqslant {}_{\max}C_k$ leads to maximise the profit function E_k.

3.6. *Agent utility*: *Price elasticity*

Generally the influence of demand factors into the demand is called 'elasticity' in economics. Price elasticity, described in Eq. (22), is one of the major factors that control economic dynamics.

$$|(\mathrm{d}x/\mathrm{d}p) \times (p/x)|. \tag{22}$$

In our market model, price elasticity, which characterises the demand function, represents agent utility for purchasing resources:

$$(\text{Price Elasticity})_{kl} = |(\mathrm{d}x_{kl}/\mathrm{d}p_i) \times (p_i/x_{kl}) = 1/(b_{kl} - 1)|. \tag{23}$$

Agent demand utility depends on b_{kl}, and that means agent demand activity affects more, as the price elasticity has a greater value in $0 < b < 1$ (refer (4)).

Suppose $R_k = r_k + 1$, then we have R_k elasticity as follows:

$$|(\mathrm{d}x_{kl}/\mathrm{d}R_k) \times (R_k/x_{kl})| = |1/(b_{kl} - 1)|. \tag{24}$$

From the comparison between (23) and (24), the reduction rate of input resource x_{kl} in the budget constraint depends on b_{kl}. Additionally, budget change affects more to the amount of demand, as the value b_{kl} increases.

3.7. *Market-oriented programming in SCM model*

In market-oriented programming, we take the metaphor of an economy computing multi-agent behaviour literally, and directly implement the distributed computation as a market price system. The algorithm of the proposed market-oriented programming in SCM is shown as follows:

Step 1: Set initial price p_i for all the resources.
Step 2: Agent k calculates x_{kl} by (20) assumed $r_k = 0$, then computes C_k by (8), (9). If $C_k > {}_{\max}C_k$ then go to *Step 3*, otherwise go to *Step 4*.
Step 3: Modify r_k followed by the Profit Maximise Theorem (Reduce r_k to satisfy $C_k = {}_{\max}C_k$).
Step 4: Define current demand/supply functions with r_k by (20), (21).
Step 5: Agent k sends the acquired demand/supply function as bids into the market to indicate its willingness to buy/sell resources.
Step 6: Market mechanism calculates the balanced prices of all resources in the competitive market.
Step 7: If all the balanced prices are sufficiently converged, then go to *Step 8*, otherwise go to *Step 2*.
Step 8: Allocate all the resources under the acquired equilibrium prices.

4. Experimental results

4.1. *Experimental model*

A basic SCM model shown in Fig. 2 is prepared to investigate the validity of the proposed approach by computer simulation. The model has a series of three-layered market structure with two-layered agent groups. This model comprises the three types of agent in each layer and three types of good. The interconnectedness of agents and goods defines the market configuration. Comparative analysis of the three market structures reveals the qualitatively distinct economic and computational behaviours realised by the proposed configurations.

Each agent has production functions to transform the resource from market ($M[i][j]$) to market ($M[i+1][j]$), and the parameters are defined as $a[j]$, $b[j]$ in (4). The parameters in each agent group are described in Table 1. The parameter b is set in common to each type of the goods, because

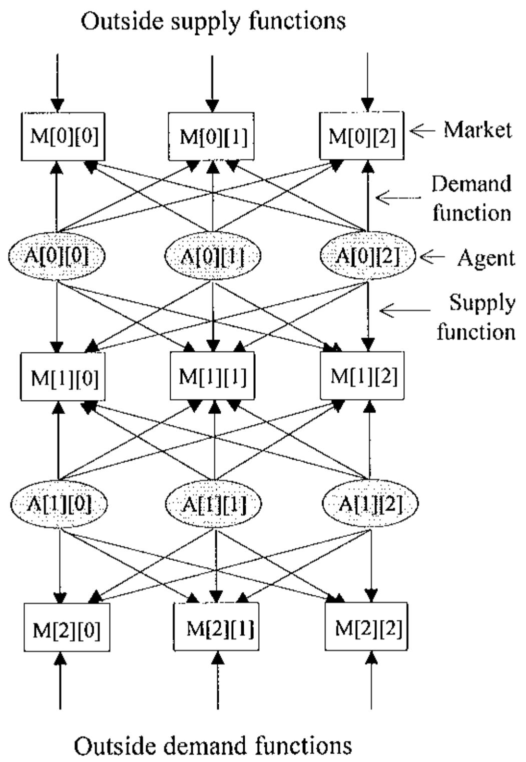

Fig. 2. SCM model.

this parameter is deeply concerned with the price elasticity of the goods shown in (23). In this figure, the outside demand function and the outside supply function, which correspond to sink and source in the experimental model, are defined respectively as

$$x_i = \alpha_i p_i^{\beta_i} \quad (\alpha_i > 0, \beta_i < -1), \tag{25}$$

$$y_i = \alpha_i P_i^{\beta_j} \quad (\alpha_j > 0, \beta_j > 0), \tag{26}$$

Table 2
Outside production function parameters of agents

	α[0]	β[0]	α[1]	β[1]	α[2]	β[2]
Supply function	100	1.5	100	1.5	100	1.5
Demand function	100	− 2	100	− 2	100	− 2

and each parameter in (25), (26) is described in Table 2.

4.2. Market dynamism and price elasticity

Dynamical changes of (i) the amount of dealing goods in supply and demand, (ii) the prices of the goods, at each layer in the market structure are shown in Figs. 3 and 4, respectively.

First of all, it is obvious that both the amount of dealings and the prices are converged into equilibrium in these figures. Since our methodology is perfectly endorsed by 'general equilibrium theory' in the competitive market, we can get a Pareto optimal solution in the equilibrium. That means the goods distribution policy followed by the acquired solutions, i.e., the amounts and the prices, are Pareto optimal in the entire market. Efficient SCM with market mechanism are attainable by the proposed approach.

Secondly, it is observed that the number of iterations required to reach equilibrium seems to rise with the price elasticity. For example, Market [0][0] with 0.7 in price elasticity takes longer time to converge than Market [0][2] with 0.3 in price elasticity in Fig. 3(a). We attribute this to the

Table 1
Production function parameters of agents

	a[0]	B[0]	a[1]	b[1]	a[2]	b[2]	Budget
Agent[0][0]	5	0.7	5	0.5	5	0.3	10
Agent[0][1]	3	0.7	8	0.5	3	0.3	10
Agent[0][2]	7	0.7	7	0.5	7	0.3	4
Agent[1][0]	10	0.3	10	0.6	10	0.4	50
Agent[1][1]	4	0.3	4	0.6	4	0.4	100
Agent[1][2]	8	0.3	4	0.6	4	0.4	6

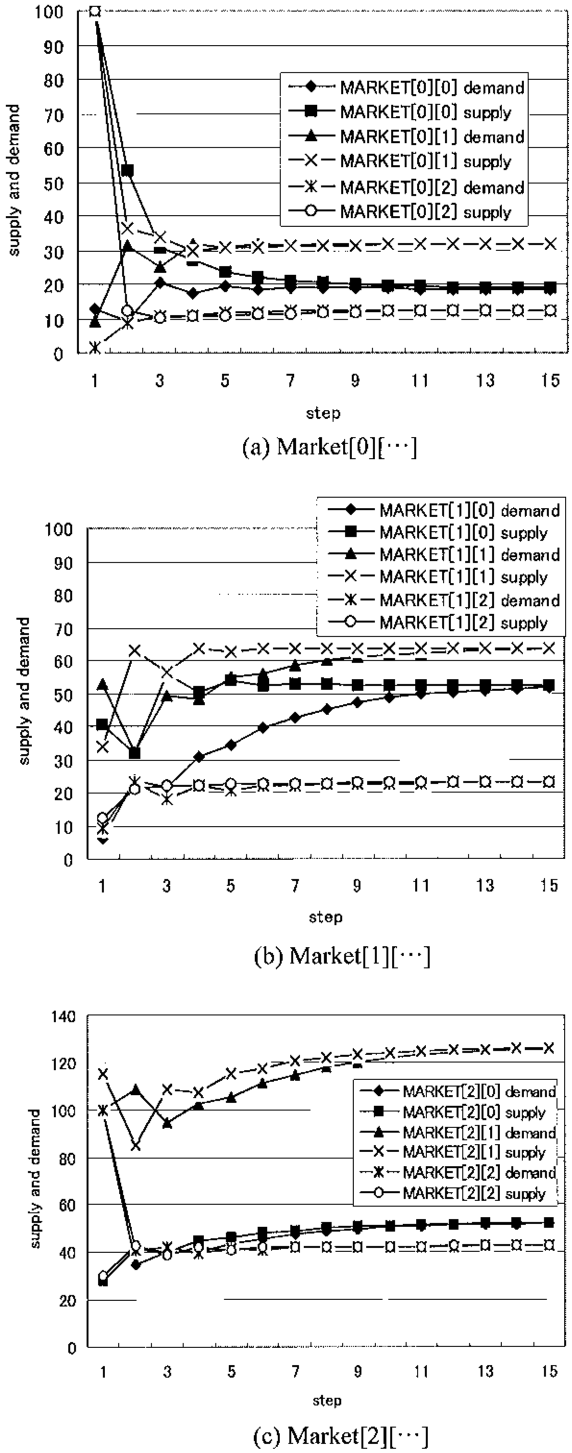

Fig. 3. Supply and demand oscillation.

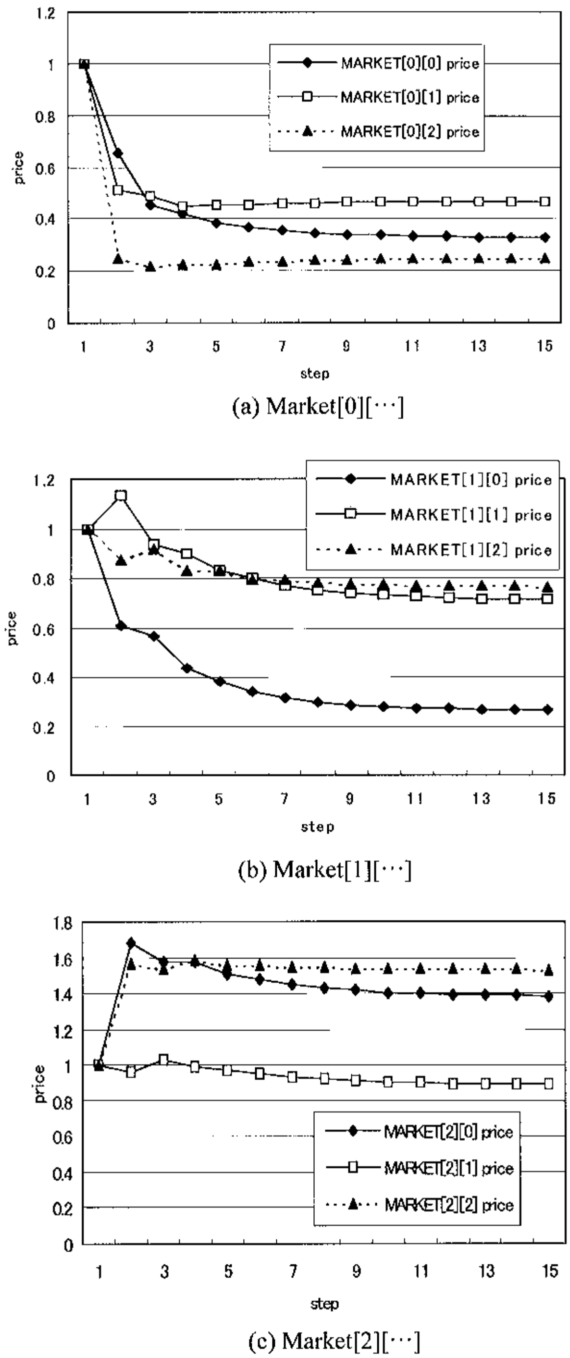

Fig. 4. Price oscillation.

natural characteristic of price elasticity formulated in (24), and the experimental values completely agree with our prior estimations described in 3.6.

Finally, it takes longer time to converge into the equilibrium at the market in the middle layer, Market [1][..], compared with the other markets, Market [0][..] and Market [2][..]. This observation is explained by the following reason. We applied the functions described in (25) and (26) as outside functions in Market [2][..] and Market [0][..], and they are defined as static functions in the experimental model. On the other hand, Market [1][..] is operated by supply and demand agents with dynamic utility functions. As a result, Market [1][..] behaves dynamically and is more sensitive to trading situation in the competitive market.

4.3. Market equilibrium

The market equilibrium dynamism should be explained by our market definition. The comparison between Figs. 3(a) and 4(a) shows us that the good with larger trading amount has higher price in the equilibrium in Market [0][..]. That is because we defined the function shown in (26) as outside supply function, which characterises positive correlation between the dealing amount and the price. On the other hand, the good with larger trading amount has lower price in the equilibrium in Market [2][..] shown in Figs. 3(c) and 4(c). The outside demand function defined in (25), which has negative correlation between the dealing amount and the price, influences the equilibrium. These experimental values are perfectly explicable by our market formulation. The middle layered market, Market [1][..] has more complex dynamism in Figs. 3(b) and 4(b). The dynamism is emerged and explained by the agent utility parameters shown in Table 1. In this case, Market [1][1] is high both on the dealing amount and on the price in the equilibrium. A set of Agent [0][..] has to offer higher sales price to increase their profit, because b[1] in Agent [1][..] (= 0.6) is the highest amongst b[..] in the second layer, but b[1] in Agent [0][..] (= 0.5) is lower than b[0] in the first layer.

It has been clarified that all the experimental values in the complex SCM model are perfectly explicable by our market formulation.

4.4. Experimental summary

We have acquired several points to validate the proposed methodology:

- A Pareto optimal solution is attainable by the equilibration process.
- The equilibration process scales with price elasticity of trading goods.
- Outside supply and demand function reduce oscillation in the equilibration process.
- The dynamism in the equilibrium highly depends on the utility functions of agents.

The experimental results agree with the theoretical trends of perfect competitive market in microeconomics. It is obvious that each market is perfectly competitive and holds market mechanism in general equilibrium.

5. Conclusions

In this paper we proposed a SCM with market economics. We formulated SCM as a distributed resource allocation system, based on general equilibrium theory and competitive mechanism. The approach works by deriving the competitive equilibrium corresponding to a particular configuration of agents and markets. After defining production functions, we introduced budget constraint for practical use and a newly proposed Profit Maximise Theorem as an agent strategy. It has been confirmed by simulation experiments that the careful constructions of the decision process according to economic principles can lead to efficient resource allocations in SCM, and the behaviour of the system can be analysed in economic terms.

The contribution of the paper lies in the idea of SCM based on market-oriented programming, an algorithm for distributed computation of competitive equilibria of computational economics, and an initial illustration of the approach on a simple supply chain model. Effective SCM in global environment is expected by this research.

Appendix A

Theorem. *Profit function E_k of agent k is maximised by minimised r_k, which satisfies the following conditions*:

$$\forall l: \quad \frac{\partial E_k}{\partial c_{kl}} = r_k \; (r_k \geqslant 0) \cap C_k \leqslant {}_{\max}C_k. \tag{A.1}$$

Proof. Note that

$$\frac{\partial f_{kl}}{\partial x_{kl}} = \frac{\partial f_{kl}}{\partial c_{kl}} \cdot \frac{\partial c_{kl}}{\partial x_{kl}} = p_i \frac{\partial f_{kl}}{\partial c_{kl}} \tag{A.2}$$

then

$$\left.\frac{\partial f_{kl}}{\partial c_{kl}}\right|_{c_{kl}=c} > \left.\frac{\partial f_{kl}}{\partial c_{kl}}\right|_{c_{kl}=c+\Delta} \quad \text{in any positive value } \Delta. \tag{A.3}$$

Also, note that

$$E_{kl} = P_j f_{kl}(x_{kl}) - p_i x_{kl} = P_j f_{kl}(c_{kl}/p_i) - c_{kl}, \tag{A.4}$$

then E_{kl} is regarded as concave down by (A.3). E_{kl} is maximised with the condition (A.5):

$$\forall l: \quad \frac{\partial E_k}{\partial c_{kl}} = 0. \tag{A.5}$$

Let C'_k denote total expense in (A.4) and if $C'_k \leqslant {}_{\max}C_k$ then the maximum profit is given with (A.5). Otherwise the maximum profit is not given with (A.5) due to the budget constraint.

If $C'_k > {}_{\max}C_k$, agent k must consider to increase C_{kl} by Δc , and reduce $C_{kl'}$ by Δc shown in (A.6), (A.7).

$$\left.\frac{\partial E_k}{\partial c_{kl}}\right|_{c_{kl}=c+\Delta c} = \frac{P_j}{p_i} f'_{kl}[(c_{kl} + \Delta c)] - 1 = r_{kl}, \tag{A.6}$$

$$\left.\frac{\partial E_k}{\partial c_{kl'}}\right|_{c_{kl'}=c-\Delta c} = \frac{P_h}{p_g} f'_{kl'}[(c_{kl'} - \Delta c)] - 1 = r_{kl'}, \tag{A.7}$$

then we obtain that $r_{kl} < r_k < r_{kl'}$ from (A.3).

Let ΔE_{kl} and $\Delta E_{kl'}$ denote the increased profit in f_{kl} and the diminished profit in $f_{kl'}$, respectively, then

$$\begin{aligned} \Delta E_{kl} &= \alpha \Delta c \; (r_{kl} < \alpha < r_k), \\ \Delta E_{kl'} &= \beta \Delta c \quad (r_k < \beta < r_{kl'}). \end{aligned} \tag{A.8}$$

It is obtained that $\Delta E_{kl} < \Delta E_{kl'}$ from (A.8), and that means the diminished profit is greater than the increased profit in any Δc.

Therefore, the profit function E_k of agent k is maximised with the condition

$$\forall l: \quad \frac{\partial E_k}{\partial c_{kl}} = r_k \; (r_k \geqslant 0) \quad \text{in } C_k \leqslant {}_{\max}C_k. \tag{A.9}$$

Finally, r_k is minimised with the condition (A.9), since the profit function E_k is concave down. This completes the proof.

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