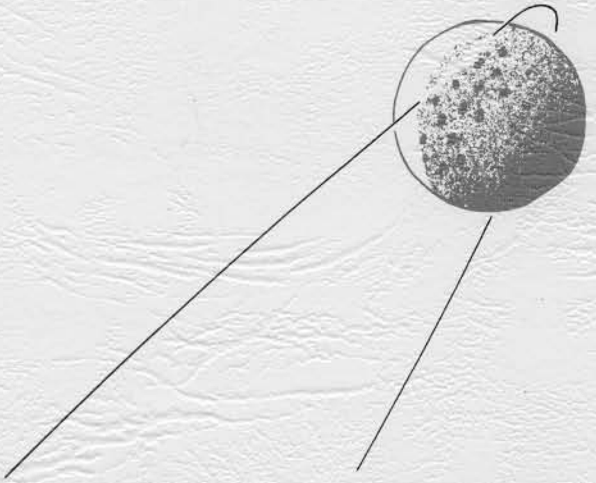


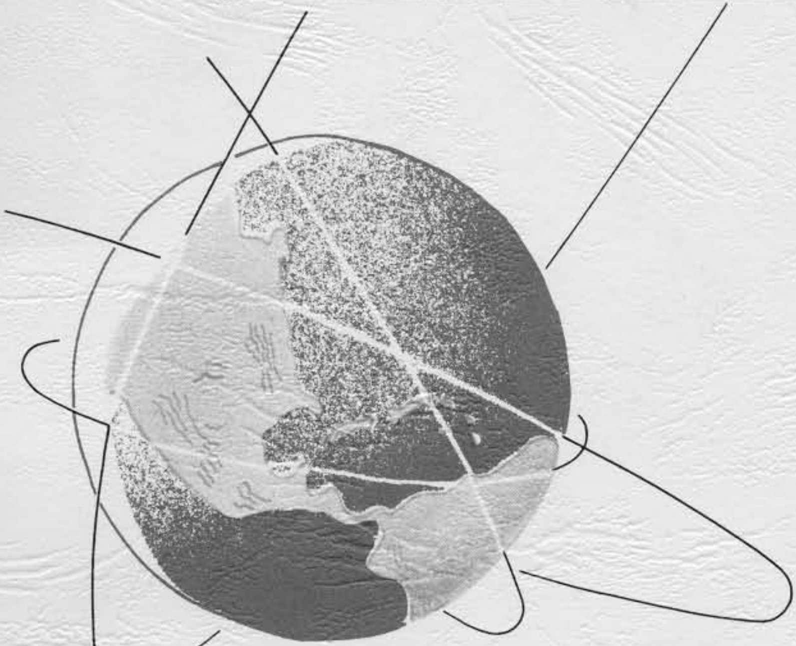
SATELLITE ORBIT PERTURBATIONS DUE TO THE GEOPOTENTIAL

by

George W. Rosborough

CSR-86-1

January 1986



CENTER FOR SPACE RESEARCH

THE UNIVERSITY OF TEXAS AT AUSTIN AUSTIN, TEXAS

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ABSTRACT

Satellite-borne radar altimeters have the capability of measuring the distance from the satellite platform to the ocean surface with an accuracy of five centimeters or less. To fully exploit the geophysical information contained in this data, it is necessary to have independent knowledge of the geocentric satellite distance to a comparable level of accuracy. Current altimeter satellite ephemerides fail this requirement by at least an order of magnitude. The radial errors contained in the estimated orbits are primarily a result of inadequacies in the modeling of the Earth's gravity field. To better understand the limitations this orbit error imposes on the analysis of the altimeter data, this study details the temporal and spatial characteristics of the radial orbit perturbations produced by the geopotential. The temporal description fully describes the frequency spectrum of the perturbations, and the spatial description provides a direct method for quantifying the perturbations on a geographical basis. These results are used to analyze the expected radial orbit perturbations of the altimeter satellite to be flown by the proposed TOPEX Mission. To facilitate other possible applications, the geopotential perturbations in the along-track and cross-track orbit directions are presented also.

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CHAPTER

INTRODUCTION

1.1 Introduction

Satellite-borne radar altimeters, as demonstrated by SKYLAB [*McGoogan et al.* 1974], GEOS-3 [*Stanley*, 1979] and SEASAT [*Tapley et al.*, 1982], are capable of providing accurate measurements of a satellite's geodetic height above the ocean surface. Given adequate temporal and spatial sampling, this altimeter data contains extensive geophysical information. This is due to the response of the ocean surface to local gravity signatures and atmospheric effects as well as global responses to tides and circulation. The value of this data has been most successfully shown by the SEASAT Mission. The relatively limited set of data acquired by the SEASAT altimeter (covering only three months in 1978) has contributed to studies of the geopotential, marine geoid, mean sea surface, tides, current variability, general circulation, bathymetry, oceanic lithosphere and tectonics. In the years since SEASAT, the bibliography of work in this area has grown considerably. Excellent collections of papers covering all aspects of satellite altimetry can be found in *Oceanography From Space, Marine Science*, Vol. 13, 1981; *Journal of Geophysical Research, SEASAT Special Issue I*, 1982; *Journal of Geophysical Research, SEASAT Special Issue II*, 1983; and *Marine Geodesy, Satellite Altimetry Theme Issue*, Vol. 8, 1984.

The full extent of geophysical information contained in the SEASAT altimeter data has yet to be realized. This is due to the systematic errors present in the modeling of the satellite motion and the modeling of the shape of the geoid. The impact of these errors varies depending on the particular geophysical phenomenon being studied. In all cases, it is important to understand how the systematic errors affect the data so that full and correct interpretation of the geophysical results can be made.

1.2 Altimeter Measurement

The altimeter flown on SEASAT provided a substantial improvement in range precision compared to the earlier GEOS-3 and sparingly used SKYLAB altimeters. The noise of the SEASAT measurements is 5 centimeters. Subsequent altimeter missions all will carry altimeters of similar or better precision [*Born et al.*, 1984]. In addition to the random noise, the measurements are affected by various systematic errors due to the instrumentation, atmosphere and ocean surface properties. Accounting for all of these error sources leads to an altimeter range accuracy of better than 10 centimeters in the case of SEASAT [*Tapley et al.*, 1982]. While it is important to recognize and account for these systematic errors in the altimeter measurement, they are currently not a limiting factor in the application of altimeter data and are not considered further here.

The primary measurement used in altimetry studies is the height of the ocean surface with respect to the Earth's reference ellipsoid. Once the altimeter measurements are suitably corrected, the ocean surface heights can be computed using a precise ephemeris that defines the satellite altitude above the reference ellipsoid for each altimeter observation time. Figure 1.1 illustrates the geometric relationships. While the systematic errors in the altimeter measurements are at the few centimeter level, the errors in the current SEASAT ephemerides are at the 50 centimeter to a meter level. Thus, the ocean surface measurements (referenced to the ellipsoid) contain significant systematic errors when they are examined either temporally or spatially.

Another altimetric measurement of interest is the height of the ocean surface with respect to the geoid. This quantity is noted as h_T in Figure 1.1. The measurements of h_T are corrupted further by deficiencies in the geoid model. However, since the geoid is a constant surface, i.e., it does not vary with time (at least at the time scales being considered here), the errors in the geoid are consistent in any particular region. That is, measurements separated in time but taken at the same location have the same geoid error.

The effect of the time-varying orbit error on the geophysical analysis is significantly more difficult to determine than the effect of the time-invariant geoid error. The principal questions addressed in this study are directed at explaining how this systematically time-varying error is mapped into an Earth-fixed frame of reference.

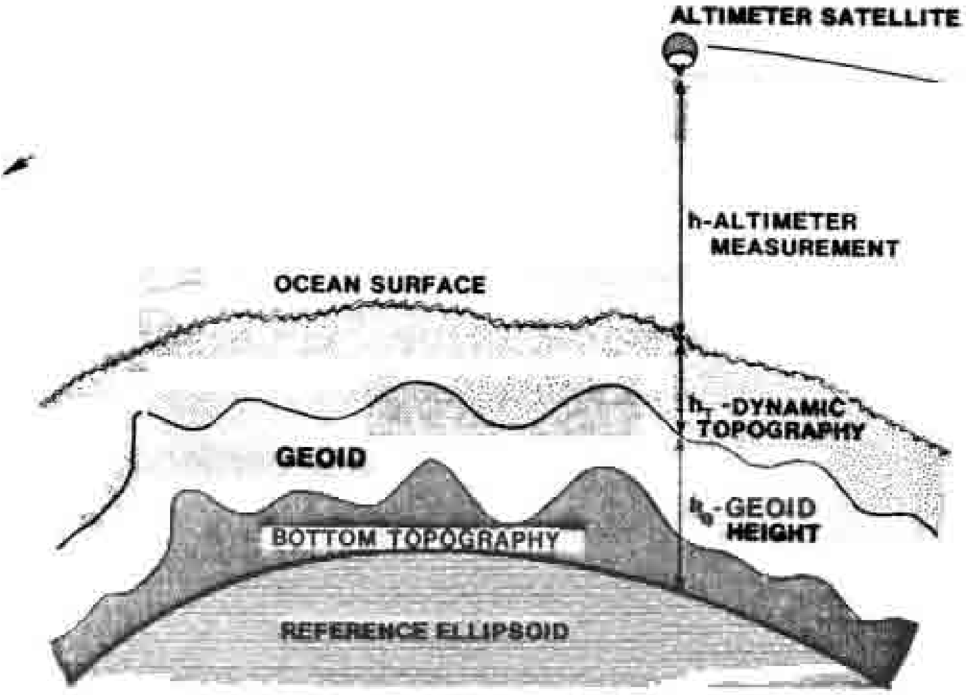

FIGURE 1.1. SATELLITE ALTIMETRY

1.3 Geoid Error

The geoid is an equipotential surface that corresponds closely to the mean sea surface. The two surfaces differ due to the effect of the ocean currents. This difference is referred to as the constant dynamic topography, and it is intimately related to the mean circulation of the oceans. While the geoid exhibits 100 meter global variations, the constant dynamic topography in most oceanic areas is less than 20 cm and only exceeds one meter where the strongest currents persist. Thus, detecting this topography from altimeter observations requires a precise mean sea surface and geoid. The accuracy of the mean sea surface is primarily dependent on the characteristics of the orbit error which are to be investigated in this study. This section will look briefly at the expected geoid accuracy based on the best model currently available.

Long wavelength (10000 km) geoid undulations have been determined accurately from ground based tracking of artificial satellites. Due to decreasing sensitivity of the satellite motion to the shorter wavelength components of the gravity field, the satellite-derived gravity models are not as precise at the intermediate (4000 km) and shorter wavelengths. Short scale (100 km) geoid features are known poorly on a global basis, although there exist various regional geoid models derived from surface gravity measurements that are reasonably precise. These models are primarily over land areas, although an accurate short wave geoid has been developed for the altimeter calibration area off the east coast of North America [*Marsh and Chang*, 1978]. Altimeter data provides a significant contribution to the determination of the geoid at all wavelengths, although the problem of separating the constant dynamic topography from the geoid remains, since both are contained in the ocean surface measurements. Simultaneous determination of the geoid and constant dynamic topography from altimeter data has yet to be fully demonstrated.

If the discussion is limited to the long and intermediate wavelength structure of the geoid, the geoid height accuracy is defined through the covariance of the geopotential coefficients used to define the geoid. The best of the long wavelength geoid models is given by the GEM·L2 gravity model [*Lerch et al.*, 1982b]. The covariance of the coefficients defining this model has been calibrated using independent data to provide a realistic measure of their accuracy [*Wagner*, 1983 and *Lerch et al.*, 1985]. The standard deviation of the geoid height defined by this model at the long wavelengths (10,000 km and longer) is shown in Figure 1.2. For features 4000 km and longer, the standard deviation is as shown in Figure 1.3. Note the appreciable degradation as the intermediate scale features are included. This wavelength dependence is summarized in Figure 1.4 which shows RMS geoid accuracy (computed globally on an even grid) as a function of wavelength.

To place this geoid accuracy in perspective, it can be compared to the expected size of the constant dynamic topography. Determinations of the constant dynamic topography based on long-term surface observations have been made for the Pacific Ocean by *Wyrtki* [1975] and globally by *Levitus* [1982]. The size of the topography signal as a function of wavelength has been determined by *Tai* [1983] using Wyrtki's model and by *Tai and Wunsch* [1984] using the Levitus model. The two results were found to be comparable. The topography signal based on the Levitus model is shown in Figure 1.4. Inasmuch as the GEM·L2 model is representative of current long wavelength geoid knowledge and the Levitus model is representative of the constant dynamic topography, then it can be concluded that only the very long wavelength topography features could be expected to be observed using an altimetric approach. Implied in this conclusion is that an accurate long wavelength mean sea surface can be determined in the presence of the orbit error. This expected observability of the large scale topography has been generally noted and several determinations have been made using SEASAT altimeter data and current geopotential models with promising results [*Cheney and Marsh*, 1982; *Tai and Wunsch*, 1983; *Douglas et al.*, 1984; *Tai and Wunsch*, 1984; *Engelis*, 1985].

So for the altimetric applications in which a geoid model is needed, it is usually possible to specify the accuracy of the model. In this way, a measure is available that indicates how much geophysical signal can be expected to be observed. A similar capability is required for the systematic effects of orbit error.

1.4 Orbit Error

Precision orbit determination efforts have produced SEASAT ephemerides that are currently accurate at the 50 to 75 cm level in an RMS sense. These orbits have been determined using ground-

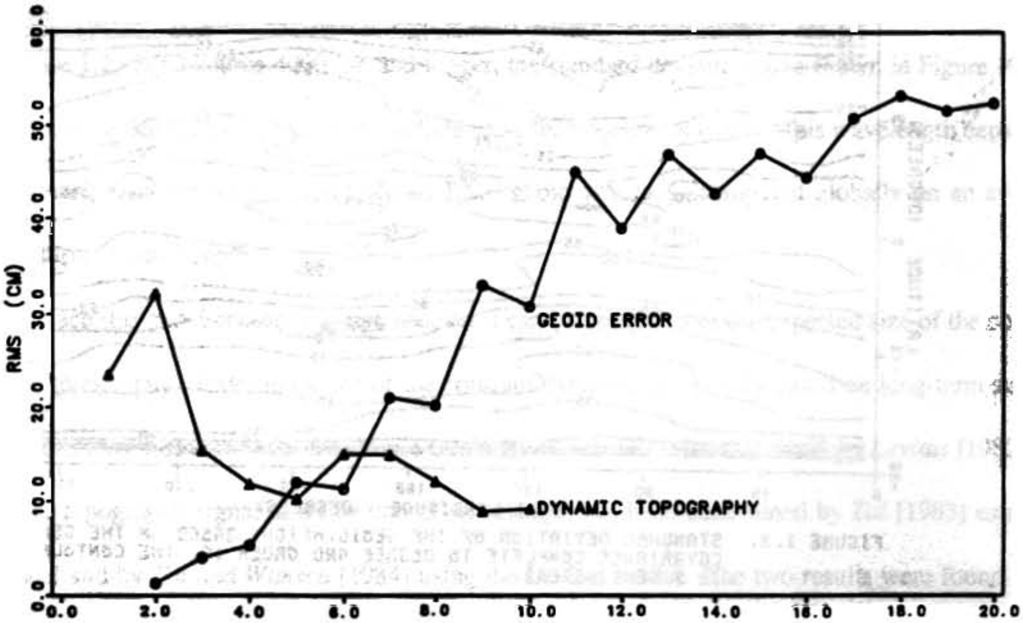

FIGURE 1.4. COMPARISON OF THE RMS OF THE GEM-L2 GEOID ERROR AND THE RMS OF THE LEVITUS DYNAMIC TOPOGRAPHY AS A FUNCTION OF DEGREE.

based laser range and Unified S-Band tracking data. The improvement in the accuracy of the orbits over the orbits released on the SEASAT Geophysical Data Records [*Lorell et al.*, 1980], is attributed to the improved SEASAT gravity model, PGS-S4 [*Lerch et al.*, 1982a], that was developed using the SEASAT tracking data together with a subset of the SEASAT altimeter data. This tailored gravity model improved the orbital accuracy from an RMS of 1.5 meters to the current level. Recently, the TRANET doppler tracking of SEASAT has been used to obtain further improvements in the SEASAT ephemerides. The use of this doppler data for gravity model improvement, where it would be most beneficial, has yet to be authorized.

The emphasis on gravity model improvement is due to the fact that geopotential modeling error remains the single largest error source in the precise computation of near-Earth satellite orbits. This remains true even as the gravity models improve, since smaller orbit errors due to geopotential mismodeling allow for better modeling of solar radiation pressure and atmospheric drag which are the next largest error sources.

The following table shows the approximate levels of orbit error due to errors in modeling the geopotential, solar radiation pressure and atmospheric drag for the current SEASAT ephemerides and the proposed TOPEX altimeter satellite [*TOPEX*, 1981].

TABLE 1.1
ALTIMETER SATELLITE RADIAL ORBIT ERROR

	RMS Radial Orbit Error (cm)	
Error Source	*SEASAT*	*TOPEX*
Geopotential	70	10
Solar Radiation Pressure	15	1
Atmospheric Drag	15	1

The significantly smaller TOPEX numbers reflect the fact that the satellite is 500 km higher than SEASAT providing significantly reduced gravity and drag perturbations. The TOPEX numbers also reflect an expected improvement in the geopotential model before the anticipated TOPEX Mission in the early 1990's.

In the case of SEASAT, it is important to understand how this relatively large geopotential-induced radial perturbation is affecting the sea surface measurements and the resulting geophysical interpretations. The question is important also for TOPEX, since the TOPEX altimeter accuracy (<5 cm) will still be better than the predicted orbit accuracy (13 cm) [*TOPEX*, 1983].

1.5 Geometric Orbit Correction

To enhance the usefulness of the SEASAT altimeter data, various methods have been devised for correcting the computed ephemerides and reducing the systematic error in the sea height measurements. The principal means for accomplishing this is by requiring consistency in the sea surface measurements. That is, two sea surface measurements at the same geographic location, separated in time, should agree in most instances to better than 20 cm. (Larger differences can be expected in the vicinity of a strong current.) In actuality, comparing such measurements usually shows meter level differences due to the orbit error. Depending on the study being performed, surface measurement consistency can be obtained in a number of ways.

Ocean current variability studies are most successfully performed using a satellite altimeter in an orbit designed to produce a repeating ground track. The repeat ground track orbit allows the altimeter to make repeated measurements along a particular geographic path that are separated by a specified time interval. Comparing the sea surface measurements along these collinear paths will show biases and systematic differences in their respective long wavelength trends. As long as the satellite repeats precisely the same path each time, then these differences are due to orbit error and time-varying ocean surface topography. Since the orbit error, which will be the predominant effect, is of a long wavelength nature, the error can be effectively removed by examining short arcs of the data and removing a bias and linear trend between the collinear track measurements. The resulting data now has no absolute reference, but the relative differences do show how the surface has changed in time. Since the arcs over which these biases and trends are adjusted must be short to insure that the orbit error is being removed, there is also no absolute relation between arcs in different regions and thus the long wavelength surface information is lost. However, for surface variability studies, this is of no consequence, and this procedure has provided excellent results using GEOS-3 [*Douglas and Cheney*, 1981] and SEASAT [*Cheney et al.*, 1983].

Another method used to reduce orbit error influences is spatial averaging. This has been used in constructing mean sea surfaces [*Marsh and Martin*, 1982]. The ocean area is gridded and all the observations available near each grid intersection are combined to obtain a mean surface height. The algorithms used to compute the mean are usually more complex than simple averaging since the distance and distribution of the data about each grid point needs to be accounted for. The amount of orbit error eliminated in this process depends entirely on the character of the orbit error affecting each point in the region. If the error affecting each point is uncorrelated with every other point, then given enough sampling, the error should average out and the effect on the constructed mean sea surface should be minimal. However, if the orbit error is correlated geographically then there will be a mapping of the correlated radial orbit error into the computed means. *Anderle and Hoskins* [1977] have demonstrated that geopotential errors can produce just such a geographically correlated orbit error. *Marsh and Martin* [1982] agree with this assessment. It is important to quantify this effect so that appropriate statistics can be applied to mean sea surfaces constructed in this manner.

A third approach to handling the systematic error is the crossing arc method. The ground trace of the satellite path along an ascending (or descending) pass will cross previously traversed descending (or ascending) ground traces. At these crossing points, the sea surface measurements obtained from the two tracks (one ascending and one descending) can be expected to agree except for the small variations due to current variability and any other unmodeled time-varying effects. (Since measurements are not usually obtained precisely at the crossing points, interpolation is used to obtain measurements to compare.) In the actual case, the discrepancy is much larger than expected due to the orbit error. The geoid error contributes equally to both measurements and is thus not a factor (except perhaps in a second-order fashion due to the interpolation).

An empirical adjustment to the orbits can then be made to reduce these crossover differences to a reasonable level. The adjustment usually consists of solving for biases and trends for short orbit segments and can be done on a regional or global basis. One difficulty with this method is specifying a reference orbit, to which the solved for biases and trends are referenced, since the crossover differences contain no absolute radial position information. For instance, one or the other of the orbits passing through the crossing could be held fixed and the other orbit could be adjusted. Since neither orbit could be expected to be of higher accuracy, the corrected observations would be more consistent but would not contain any less orbit error. For local regions this may not be a problem if the absolute height measurements are not of interest. Another method is to combine collinear tracks and compute an average orbit and then reference the corrections to the average [*Rowlands*, 1981]. The effectiveness of this method depends on how well the averaging removes the systematic orbit errors. Clearly, if the orbit error is geographically correlated, the reference orbit will still contain some of the orbit error. A third approach is to require all the estimated biases in a particular region to average to zero [*Parke and Stavert*, 1985]. Again, the resulting sea surface measurements will still be corrupted by that component of the orbit error that is geographically correlated.

Yet another approach is to use the altimeter data to provide an absolute reference. This technique is favorable in that it avoids the problem of correlated orbit error; however, it also requires that a sufficiently precise geoid model exist as a reference for the altimeter data. *Douglas et al.* [1984] used this technique to solve for the constant dynamic topography with only a three-day set of SEASAT altimeter data. They note that the results were not unduly affected by the choice of geoid model.

Despite the problems that exist when using these crossing arc techniques, they are effective in producing a more consistent sea surface data set. The RMS of the crossovers from the best dynamically computed ephemerides is approximately one meter. The crossover RMS after geometrically correcting the orbit by any of these methods is typically 20 cm which is consistent with the expected variability in the ocean surface. The RMS values in either case vary significantly depending on the region being examined and the time interval considered. Though the stated numbers show the type of increase in measurement consistency provided by the crossover methods.

To maximize the geophysical knowledge gained from these improved measurements, it is still necessary to understand the nature of the radial orbit error. Provided this, it is then possible to determine how well these various methods are reducing the systematic error. More importantly, it is then possible to apply realistic measures of accuracy to the corrected data which can then be used to determine appropriate statistics for the estimated geophysical quantities.

1.6 Study Objectives

The existing GEOS-3 and SEASAT altimeter data will certainly be supplemented by further planned and proposed altimetry missions over the next decade [*Born et al.*, 1984]. Analysis of these data to produce geophysical parameters needs to be accompanied by an analysis of the systematic errors in the data so that appropriate statistics can be attached to the results.

The largest error source in sea surface measurements is the uncertainty in the computed satellite ephemerides based on independent tracking. The difficulty in providing an accurate ephemeris (commensurate with the altimeter accuracy) is predominantly due to inadequate modeling of the geopotential. The objective of this study is to characterize the radial orbit error due to geopotential error, both temporally and spatially.

A temporal representation provides the spectrum of the radial errors. This can be used to quantify the shortest periodicities of the radial error that cause significant perturbations. This then puts a limit on the size of small scale phenomenon that can be studied without aliasing from the orbit error. The temporal representation also can be used to determine the frequencies of the largest errors. These frequencies could then be used in attempts to empirically remove orbit error. Knowing the functional form of the radial error also makes it possible to use the geopotential coefficient covariance (specifying the accuracy and correlations of the coefficients) to determine the expected radial orbit accuracy. This provides a needed statistic that can be applied to the sea surface measurements which can then be transformed to the required statistics for the geophysical result.

Transforming the temporal representation of the radial error to a form that is explicitly a function of the satellites body-fixed location provides a spatial representation of the error. This approach to studying the radial error provides great insight into how these geopotential-induced perturbations are manifested in the body-fixed frame. And provides a valuable tool for studying how orbit error impacts altimetric measurements. Most non-dynamic methods used to improve the orbits are very sensitive to geographically correlated orbit error. The spatial representation of the error addresses this problem directly. As such, the amount of geographic correlation can be quantified and the impact on the corrected measurements can be obtained.

To provide a clear understanding of the nature of the radial effect derived from these representations, the radial perturbations that will effect the proposed TOPEX orbit will be fully evaluated both temporally and spatially. Also, using the best available measure of geopotential modeling accuracy, the expected radial accuracy of the TOPEX orbit will be determined. The level of this radial orbit accuracy will give an indication of the improvement in geopotential knowledge required for the TOPEX Mission to meet its goals.

While this study specifically addresses the question of radial orbit error with particular emphasis on satellite altimeter missions, the analysis is general in nature and should find application to other problems in satellite orbit determination. A case in point would be the estimation of station coordinates based on artificial satellite tracking. A geographic correlation of the orbit error will definitely affect these results. Accounting for the effect will lead to better statistical properties for the estimate and thus improve its geophysical value.

1.7 Description of Chapters

The dissertation is effectively divided into two parts. Chapters 2, 3 and 4 cover the temporal representation of the radial error due to geopotential modeling error. The complementary part is the spatial representation of the error and it is presented in Chapters 5, 6 and 7.

The derivation of a linear formulation for the radial error as an explicit function of time is given in Chapter 2. This chapter also contains a brief description and presentation of Kaula's solution for the orbit element perturbations due to the geopotential. The radial perturbation is derived directly from Kaula's solution and is obtained in a completely analogous form. (Representations of the geopotential perturbation in the transverse and normal components are included for completeness in Appendix A.) Chapter 3 uses this result to characterize the radial error in the frequency domain and also as a function of the degree and order of the geopotential coefficients. The RMS of the radial perturbation is used to quantify the effect. The perturbations acting on TOPEX due to the geopotential as well as those due to an error field obtained by differencing two current models is given also. The final chapter on the temporal representation demonstrates how the geopotential covariance can be mapped into a radial variance using the linear formulation. The covariance used represents the current state of the art in long wavelength geopotential modeling. This provides a realistic measure of how accurately the TOPEX orbit could currently be expected to be computed.

The chapters on the spatial form of the radial error follow the same pattern as Chapters 2, 3 and 4. The derivation of the radial error as a function of the satellite geographic position is given in Chapter 5. Chapter 6 uses this result to characterize the spatial nature and geographic correlation of the orbit error. Again, the TOPEX satellite orbit is used for illustration. The effect of the geopotential and an assumed geopotential error both are presented. (The relation between orbit error and geoid error is illustrated with some supplementary figures to this chapter in Appendix B.) The mapping of the geopotential covariance into the radial component as a function of the body-fixed satellite location is given in Chapter 7. This provides the means for specifying the accuracy of the orbit regionally. Also, due to the form of the spatial solution, the uncertainty in the geographically correlated component can be determined. In this way, the accuracy of sea surfaces that are affected by geographically correlated orbit error can be specified. The TOPEX orbit is used as an application to discuss these points.

A brief summary of the results from the analysis introduces Chapter 8. Then conclusions that can be drawn from this study are presented. Finally, recommendations for applying these results to the analysis of existing altimeter measurements are put forth.