

# Microgrid Economic Dispatch with Energy Storage Systems

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**Abstract**—This paper presents a formulation to determine the appropriate power dispatch of an energy storage system, whose available energy is dependent on the charging/discharging pattern from previous time periods. The implementation structure is consistent with current dispatch algorithms used in microgrids, and the algorithm can be used in either grid-connected or islanded modes of operation. The proposed approach employs a backcasting algorithm to estimate the net stored energy value, against which the current cost of energy is compared to determine how the storage system should be used to perform arbitrage. The contribution of this work is a means to include the time-dependent resource in traditional economic dispatch algorithms to reduce the cost of energy in a microgrid while enabling the arbitrage algorithm to continuously adapt to changing market conditions. Results show that the backcasting algorithm is able to reduce the average cost of energy by 8.14% and can reduce the average cost of energy by up to 72.3% of the ideal reduction, as determined by a perfect forecasting dispatch.

## NOMENCLATURE

The following variables and parameters are used throughout the paper. Boldface denotes vectors of variables, while capitals denote system parameters.

$\alpha_{ch}$	Derived quadratic coefficient for the ESS' charging cost curve [\$/kW <sup>2</sup> ]
$\alpha_{dis}$	Derived quadratic coefficient for the ESS' discharging cost curve [\$/kW <sup>2</sup> ]
$\beta_{ch}$	Derived linear coefficient for the ESS' charging cost curve [\$/kW]
$\beta_{dis}$	Derived linear coefficient for the ESS' discharging cost curve [\$/kW]
$E_{ESS}$	Energy Rating of ESS [kWh]
$e_{ESS}$	Energy stored in ESS [kWh]
$\Delta e_{int}$	Change of internal energy stored in ESS [kWh]
$\eta_{ch}$	charging efficiency of ESS [%]
$\eta_{dis}$	discharging efficiency of ESS [%]
$f(\cdot)$	Probability density function
$F(\cdot)$	Cumulative distribution function
$I_{ESS}$	ESS charging/discharging current [A]
$\mathbf{J}_{n,m}$	$n \times m$ matrix of ones

$P_{EPS}$	Power limit of the transformer at the point of common coupling [kW]
$P_{ESS}$	Power Rating of ESS [kW]
$p_{ESS}$	Power output of ESS [kW]
$p_{ESS,ch P}$	ESS charging power magnitude based on cost function [kW]
$p_{ESS,dis P}$	ESS discharging power magnitude based on cost function [kW]
$p_{ESS,max}$	Maximum power of ESS [kW]
$p_{ESS,min}$	Minimum power of ESS [kW]
$P_{load}$	Power consumption of load [kW]
$P_{loss}$	Power losses in ESS [kW]
$p_{max,ch C}$	Maximum charging power of ESS based on available capacity [kW]
$p_{max,ch \eta}$	Maximum ESS charging power magnitude based on efficiency [kW]
$p_{max,dis C}$	Maximum discharging power of ESS based on available energy [kW]
$p_{max,dis \eta}$	Maximum ESS discharging power magnitude based on efficiency [kW]
$p_{min,ch E}$	Minimum ESS charging power magnitude based on ESS capacity [kW]
$p_{min,dis E}$	Minimum ESS discharging power magnitude based on ESS capacity [kW]
$\pi$	Price of energy [\$/kWh]
$\pi_{ch}$	Price of energy when it was charged [\$/kWh]
$\pi_{dis}$	Price of energy when it was discharged [\$/kWh]
$\pi_{mean}$	Moving average of the price of energy from the past $T$ hours [\$/kWh]
$\boldsymbol{\pi}_T$	Vector of costs from the previous $T$ hours [\$/kWh]
$\Delta\pi$	Difference between marginal cost and mean cost [\$/kWh]
$\Delta\pi_{ch}$	Difference between mean cost and marginal cost for charging function [\$/kWh]
$\Delta\pi_{dis}$	Difference between marginal cost and mean cost for discharging function [\$/kWh]
$\Delta\boldsymbol{\pi}_{T,ch}$	Vector of cost differences for the charging function [\$/kWh]
$\Delta\boldsymbol{\pi}_{T,dis}$	Vector of cost differences for the discharging function [\$/kWh]
$R_{ESS}$	Effective ESS resistance [ $\Omega$ ]
$T$	Time duration used for cost analysis [h]
$t_0$	Present time period [h]
$\Delta t$	Dispatch time interval [h]
$V_{dc}$	Internal ESS voltage [V]

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## I. INTRODUCTION

**T**IME-Dependent Resources (TDR), such as Energy Storage Systems (ESS), have been proposed as key technologies to be included in microgrids in order to attain certain benefits such as improving power quality, smoothing power fluctuations from renewables, improving reliability, and reducing the average price of energy [1]. An established method for solving the economic dispatch in a microgrid is by solving the Lagrangian function of the generators based on their price versus power output curves [2]. This formulation, however, is time independent and therefore does not accommodate TDRs. In order for the TDR to be included in the Economic Dispatch (ED), the resource must either be formulated as a time independent resource, or the ED formulation must be modified. In both cases, however, the Net Stored Energy Value (NSEV) in the TDR must be determined and quantified, and is based on its value as compared to the value of energy at different time periods.

This is done either through arbitrage when the microgrid can either buy or sell energy on the energy market [3, 4] or net metering with Time-of-Use (TOU) pricing [5], or when in islanded mode to operate the Distributed Energy Resources (DER) near their optimized power set-point [6]. However, since an ESS is neither a net producer nor consumer of energy, its available energy is dependent on how it was used in the past, and it must account for provisions into how it will be used in future time periods.

Although no energy is produced by such resources, TOU and market pricing structures for electric energy provides a means for a TDR to gain a net economic profit through arbitrage [7]. Since ESSs are neither net generators nor loads, they gain their economic value by applying the principle of “buy low, sell high” [8]. However, in order to utilize a TDR in an economic dispatch and maximize its NSEV, knowledge of the pricing structure and provisions for future states is required to appropriately determine an optimized dispatch schedule over multiple time steps [9].

Most studies in the literature employ a forecasting algorithm to predict future energy pricing, load profile, and generation availability from stochastic resources. A vast majority of Energy Management Systems’ (EMS) dispatch algorithms depend upon forecasting that are assumed to be 100% accurate over the future dispatching interval [3, 5, 10, 11, 12]. Although significant work has gone into forecasting algorithms [9], this deterministic approach to future dispatching does not consider the effects of forecasting errors, which can lead to sub-optimal dispatch solutions [8]. In terms of forecasting errors, it has been stated that even some of the best commercially available wind forecasting algorithms are prone to standard deviation accuracies as much as 15-20% [13, 14], with prediction errors up to 204% on an abnormal day [9].

Authors of [15] attempt to mitigate the errors by implementing a second dispatching algorithm to the ideal forecasted scheduler to adjust the real-time dispatch in order to account for the forecasting errors. In [16], authors take a conservative approach and determine a worst-case transaction cost based on the unknown stochastic variables in the optimization, used

as a bound for the solution when optimizing the utility function of the dispatchable DER. Microgrid operating costs are minimized in [17] through a niching evolutionary algorithm to use the ESS to operate the other DER close to their optimal operating point; however, no provisions for future energy use from the ESS are taken into consideration.

Stochastic ED techniques have been suggested by many authors to address the uncertainties in microgrids—predominantly to address volatile renewable generation, but also for fluctuating energy markets. To feed the stochastic ED, authors of [18] propose a Markov-chain forecasting approach based on historical data, while authors of [19] aim to reduce the errors by proposing a means to better quantify the uncertainty. Many of the proposed approaches break down the problem into different timescales [20]. Several authors propose a two-stage stochastic optimization, whereby the first stage of the ED performs the stochastic optimization based on the expected forecast, and the second stage attempts to mitigate any discrepancies by reacting to the deviations as they occur [21, 22, 23]. Although many of the stochastic methods demonstrate favourable results compared to deterministic approaches, often their solutions either require a high level of complexity in the formulation, are not adaptable to changes in the stochastic distribution of the fluctuating variables, and/or attempt to mitigate the errors *a posteriori*.

An interesting approach, suggested in [10], is to presume that the near future price structure is similar to the recent past prices (instead of implementing a forecasting algorithm). This “backcasting” approach estimates future price trends based on the previous two weeks of data, although insufficient detail is provided regarding the employed algorithm. Furthermore, this approach considers the ESS’ charging and discharging efficiencies as constant values, and it does not consider the effects of varying the ESS’ size.

This paper enhances the approach mentioned in [10] to detail a methodology that can determine an appropriate NSEV for the TDR such that it can be directly applied within the framework of the classical ED problem. This is relevant as the microgrid controller must determine the dispatch of all DER in the microgrid in both grid-connected and islanded modes of operation, and this approach eliminates the need for a forecasting algorithm or an alternate method to incorporate a TDR in the economic dispatch. The limits of the TDR must be considered in the formulation of the relative NSEV as this will limit how the energy can be used presently or in a later time frame.

Although many papers include a variety of features to model storage systems, specifically for battery ESSs, the three main characteristics that are used for its modelization include: power rating, energy capacity, and charging/discharging efficiencies [10]. Another variable operating characteristic is the State-of-Charge (SOC) of the ESS, as this determines the extent to which the ESS can be used at a later time period. Although some papers implement SOC limits to ensure sufficient reserves [25, 26], other approaches use an ideal SOC as a metric in the optimization [27]. This paper addresses each of these operating limitations in the formulation in order to best utilize

the available TDR<sup>1</sup>.

This paper is structured as follows: Section II describes the formulation of the ESS' dispatch strategy for arbitrage (Section II-A), which is limited by the ESS' efficiency (Section II-B) and stored energy (Section II-C). The final summarized dispatch is presented succinctly in Section II-D. The system description is presented in Section III, and case study results and analyses are detailed in Sections IV-V, respectively.

## II. DISPATCH STRATEGY FORMULATION

The value of energy from a TDR for arbitrage purposes must not only consider current energy prices, but provisions for future energy prices as well as its current state of charge. The dispatch of the ESS is dependent on three technical limits: its charging/discharging efficiency, its capacity, and its power rating; however, the acquisition and resale prices of the energy stored will be a primary factor in how the ESS is to be utilized to maximize profit.

### A. Dispatch based on Price Function

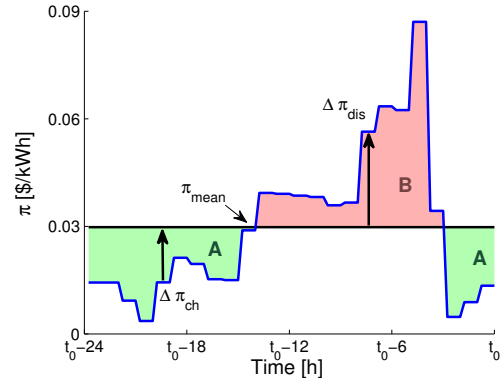
Since the economic gain from an ESS is based on past, present, and future prices of energy, the dispatch of the ESS is determined from a comparison of the current price,  $\pi(t_0)$ , and the prices in the past  $T$  hours. The presumption here is that the trend of the energy price of the past  $T$  hours is consistent with the next time step interval for the dispatch. Therefore, it is solely based on known previous time steps, and no forecasting algorithm is required.

The strategy is to charge the ESS when the price is low, and to discharge when the price is high; however, these are both relative and qualitative terms. The mean price ( $\pi_{\text{mean}}$ ) is thus calculated from the past prices in  $T$  to have a reference for the present time frame; against which the prices can be compared and determined to be expensive or inexpensive, as shown in Fig. 1.

Then, in order to determine how much the ESS should charge/discharge, one must determine how often the price of energy will be less or more expensive than the current price. Consider the following example with two scenarios: if at time  $t_0$ , the price of energy is 0.03 \$/kWh above  $\pi_{\text{mean}}$ ,

- 1) and the price from the past  $T$  hours indicates that it typically reaches a maximum of 0.03 \$/kWh above  $\pi_{\text{mean}}$ , then the ESS should be discharged at its maximum power rating.
- 2) but the price from the past  $T$  hours indicates that it can reach up to 0.06 \$/kWh above  $\pi_{\text{mean}}$ , then it would not

<sup>1</sup>The lifetime has been considered as a characteristic of certain battery storage technologies in some studies [5, 7]. However, as [24] indicates, this is an auxiliary objective to the direct price minimization objective. Therefore, this is neglected in this study to focus on the most pertinent features of the storage system that have a direct effect on the usable energy that is being charged and discharged from/to the microgrid. Furthermore, the depth of discharge limitation is considered as a hard constraint with  $E_{\text{ESS}}$  representing the net effective capacity of the ESS after incorporating this constraint. In fact, one could also reassess this value on a daily basis given how the asset was used in the previous 24 hours. Moreover, if it is perceived that the ESS lifetime is going to be shortened in light of the ESS use, the operator has the flexibility of further limiting the depth of discharge by increasing the minimum energy level in follow-up planning cycles.



**Fig. 1:** Example of market price of electricity over  $T = 24$  h (April 21, 2014). Green areas marked “A” indicate that the price is below the mean price (inexpensive), while red areas marked “B” indicate that the price is above the mean price (expensive).

be best to discharge at maximum power rating until the price is greater than the current price.

Therefore, to determine how often the difference the price of energy,  $\Delta\pi$ , is less than/greater than  $\pi_{\text{mean}}$ , two cumulative distribution functions (CDFs) are established for the price being below or above the mean price of energy, respectively. Further details and the formulation are provided in subsequent paragraphs, and Fig. 2 demonstrates an example of converting the 24-hour price profile from Fig. 1 into respective CDFs.

Given the vector of the past marginal prices of energy:

$$\pi_T(t_0) = [\pi(t_0 - \Delta t) \ \pi(t_0 - 2\Delta t) \ \dots \ \pi(t_0 - T)]', \quad (1)$$

let the vector of differences between the current price of energy and the mean price of energy from the past  $T$  hours be:

$$\Delta\pi_{T,\text{ch}} = \{\mathbf{J}_{T,1} \cdot \pi_{\text{mean}} - \pi_T \mid \pi_T < \pi_{\text{mean}}\} \quad (2)$$

$$\Delta\pi_{T,\text{dis}} = \{\pi_T - \mathbf{J}_{T,1} \cdot \pi_{\text{mean}} \mid \pi_T > \pi_{\text{mean}}\}. \quad (3)$$

These empirical histograms are shown graphically in the example shown in Fig. 2a and Fig. 2b.

The Probability Density Function (PDF) is employed to identify the relative probability that the price will take a certain value. The PDFs of the relative inexpensive and expensive price differences are defined as:

$$f_{\text{ch}}(\Delta\pi_{T,\text{ch}}) = \{f(\Delta\pi_T) \mid \pi_T < \pi_{\text{mean}}\} \quad (4)$$

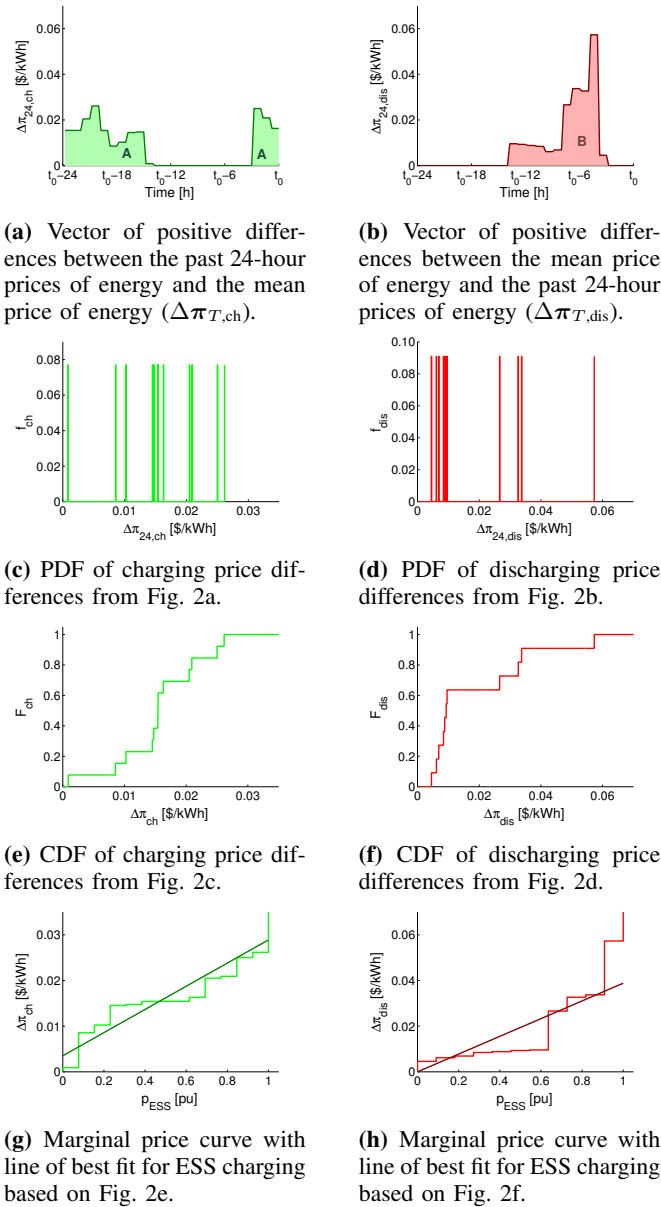
$$f_{\text{dis}}(\Delta\pi_{T,\text{dis}}) = \{f(\Delta\pi_T) \mid \pi_T > \pi_{\text{mean}}\}, \quad (5)$$

respectively. The two functions are separated to ensure that  $f > 0$ . The charging and discharging PDFs of the example from Fig. 1 are shown in Fig. 2c and Fig. 2d. The real market data used in the example have an accuracy of  $10^{-4}$  \$/kWh; thus, each price has the same probability in the PDFs since they are all unique.

The CDF of the charging function is formulated as:

$$F_{\text{ch}}(\Delta\pi_{\text{ch}}) = \Pr[\Delta\pi_{T,\text{ch}} \leq \Delta\pi_{\text{ch}}] = \int_0^{\Delta\pi_{\text{ch}}} f_{\text{ch}}(C) \, dC \quad (6)$$

In other words, (6) provides the probability that the present price difference  $\Delta\pi_{\text{ch}} = \pi_{\text{mean}} - \pi(t_0)$  takes on a value in the



**Fig. 2:** Example results of calculating the PDF, CDF, and marginal price curves for a  $T = 24$  h time frame.

range  $\Delta\pi_{T,ch} \in [0, \Delta\pi_{ch}]$ , based on an analysis of the prices of energy from the past  $T$  hours. Similarly, the CDF of the discharging function is formulated as:

$$F_{dis}(\Delta\pi_{dis}) = \Pr[\Delta\pi_{T,dis} \leq \Delta\pi_{dis}] = \int_0^{\Delta\pi_{dis}} f_{dis}(C) dC \quad (7)$$

where (7) provides the probability that the price difference  $\Delta\pi_{dis} = \pi(t_0) - \pi_{mean}$  from the past  $T$  hours (when the price is greater than the mean price) is within the range  $\Delta\pi_{T,dis} \in [0, \Delta\pi_{dis}]$ . The CDFs of the charging and discharging functions are shown in Fig. 2e and Fig. 2f, respectively. In summary, the CDFs provide an indication of the likelihood that the price of energy will become less/more expensive in a later time period (in which case, the ESS should save energy to charge/discharge at that time), or whether it is relatively inexpensive/expensive,

in which case it should charge/discharge to its maximum capabilities.

Therefore, by multiplying the CDF by the power rating of the ESS, one can determine the power that the ESS should charge/discharge based on the vector of past marginal prices:

$$p_{CDF,ch}(\Delta\pi_{ch}, t_0) = P_{ESS} \cdot F_{ch}(\Delta\pi_{T,ch}(t_0)) \quad (8)$$

$$p_{CDF,dis}(\Delta\pi_{dis}, t_0) = P_{ESS} \cdot F_{dis}(\Delta\pi_{T,dis}(t_0)) \quad (9)$$

This is sufficient to determine the appropriate amount of power to dispatch to the ESS; however, a further step is taken in order to make the NSEV compatible with standard economic dispatch algorithms (in either grid-connected or islanded modes of operation for a microgrid). This is done by formulating the power curve as a thermal resource with a quadratic cost curve and a linear marginal cost curve.

Since (8) and (9) are one-to-one functions, the CDF functions can be inverted to get the marginal price as a function of the power. Then, a curve fitting approach is used to determine the linear marginal price of energy versus power curve, as shown in Fig. 2g and Fig. 2h. This results in marginal price curves of the form:

$$\Delta\pi_{T,ch}(p_{ESS,ch}, t_0) = 2\alpha_{ch}p_{ESS,ch} + \beta_{ch} \quad (10)$$

$$\Delta\pi_{T,dis}(p_{ESS,dis}, t_0) = 2\alpha_{dis}p_{ESS,dis} + \beta_{dis} \quad (11)$$

Note that these can easily be arranged back as:

$$p_{ESS,ch|P}(\Delta\pi_{T,ch}, t_0) = \frac{\Delta\pi_{ch}(t_0) - \beta_{ch}}{2\alpha_{ch}} \quad (12)$$

$$p_{ESS,dis|P}(\Delta\pi_{T,dis}, t_0) = \frac{\Delta\pi_{dis}(t_0) - \beta_{dis}}{2\alpha_{dis}} \quad (13)$$

Although it is not directly addressed in this paper, the added benefit of this last step is that it facilitates the integration of the TDR into the classical ED problem as its price curve is in the same quadratic form as traditional thermal generators. This formulation not only permits the TDR to know its appropriate power output as a function of the price of energy at a given time period, but it can also be used in the economic dispatch for the microgrid's islanded mode of operation when the load is not curtailed.

### B. Dispatch limit based on efficiency

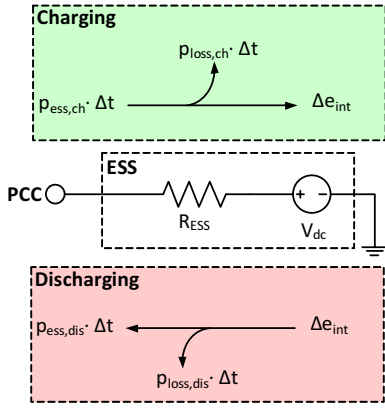
Many papers treat the efficiencies as constant values in their studies [28]. However, many authors identify that this approach is inaccurate and can introduce errors in the optimization [12, 29, 30]; here, it calculated as a function of the  $I^2R$  losses as the charging or discharging rate varies.

The energy flow in an ESS is shown through the circuit diagram in Fig. 3. Referring to this figure, the energy stored when charging the ESS is:

$$\Delta e_{int} = \eta_{ch} \cdot p_{ESS,ch} \cdot \Delta t, \quad (14)$$

where  $\eta$  is the efficiency of the ESS and  $\Delta t$  is the time period. The energy extracted from the ESS when discharging is:

$$\Delta e_{int} = \frac{p_{ESS,dis}}{\eta_{dis}} \Delta t. \quad (15)$$



**Fig. 3:** Energy flow through the ESS, where  $V_{dc}$  is the internal dc voltage and  $R_{ESS}$  is the effective resistance between the internal voltage source and the point of connection with the distribution lines on the  $V_{dc}$  base.

In order to obtain a profit and perform arbitrage for a given time step, the value of the energy when discharging must be greater than the value of the energy when it was charged:

$$p_{ESS,dis} \cdot \Delta t \cdot \pi_{dis} > p_{ESS,ch} \cdot \Delta t \cdot \pi_{ch} \quad (16)$$

Note that (16) is not a hard constraint, but it is a desired condition of the operation that would result in a net economic gain if satisfied.

For a round-trip charging of internal energy  $\Delta e_{int}$ , rearranging (14) and (15) and substituting into (16) yields:

$$\Delta e_{int} \cdot \eta_{dis} \cdot \pi_{dis} > \frac{\Delta e_{int}}{\eta_{ch}} \cdot \pi_{ch} \quad (17)$$

$$\frac{\pi_{dis}}{\pi_{ch}} > \frac{1}{\eta_{ch} \eta_{dis}} \quad (18)$$

This means that the ratio of the price of energy when discharging relative to the price of energy when it was charged must be greater than the inverse of the round-trip efficiency in order to make a financial gain from arbitrage.

In order to isolate the charging and discharging requirements, (18) is modified to include a mean price term:

$$\frac{\pi_{dis}}{\pi_{mean}} \cdot \frac{\pi_{mean}}{\pi_{ch}} > \frac{1}{\eta_{ch}} \cdot \frac{1}{\eta_{dis}} \quad (19)$$

such that

$$\pi_{ch} \leq \pi_{mean} \leq \pi_{dis}. \quad (20)$$

Note that if  $\pi_{ch}$ ,  $\pi_{dis}$ ,  $\pi_{mean}$ ,  $\eta_{ch}$ , and  $\eta_{dis}$  are all positive, and if both (21) and (22) are true, then it can be shown that (19) is satisfied, and thus (18) is also satisfied.

$$\frac{\pi_{mean}}{\pi_{ch}} > \frac{1}{\eta_{ch}} \quad (21)$$

$$\frac{\pi_{dis}}{\pi_{mean}} > \frac{1}{\eta_{dis}} \quad (22)$$

This allows for the independent calculations of the ESS' charging and discharging power limits based on the efficiencies. Although  $\pi_{mean}$  can be chosen arbitrarily, here it will be chosen as the mean price from the past 24-hour period, as this

time duration is most likely to capture any diurnal patterns in the pricing profile.

If one were to take the efficiency as a constant, (21) and (22) would suffice to determine when to charge and discharge the ESS based on the round-trip efficiency and pricing signal. However, with a battery ESS, the efficiency is a function of the power being charged or discharged (i.e.,  $\eta = \eta(p_{ESS})$ ) since it is a factor of the effective  $I^2 R$  losses.

The charging efficiency is given by:

$$\eta_{ch}(p_{ESS,ch}) = \frac{p_{out}}{p_{in}} = \frac{p_{in} - p_{loss}}{p_{in}} = \frac{p_{ESS,ch} - p_{loss,ch}}{p_{ESS,ch}} \quad (23)$$

where the discharging losses are:

$$p_{loss,ch} = I_{ESS}^2 R_{ESS} \cdot 10^{-3} = \left( \frac{p_{ESS,ch}}{V_{dc}} \right)^2 R_{ESS} \cdot 10^3. \quad (24)$$

The factor of  $10^3$  is included to ensure that all power values are given in kilowatts. Substituting (24) into (23) yields:

$$\eta_{ch}(p_{ESS,ch}) = 1 - p_{ESS,ch} \left( \frac{R_{ESS} \cdot 10^3}{V_{dc}^2} \right). \quad (25)$$

Similarly, the discharging efficiency is given by:

$$\eta_{dis}(p_{ESS,dis}) = \frac{p_{out}}{p_{in}} = \frac{p_{out}}{p_{out} + p_{loss}} = \frac{p_{ESS,dis}}{p_{ESS,dis} + p_{loss,dis}} \quad (26)$$

where the discharging losses are:

$$p_{loss,dis} = I_{ESS}^2 R_{ESS} \cdot 10^{-3} = \left( \frac{p_{ESS,dis}}{V_{dc}} \right)^2 R_{ESS} \cdot 10^3. \quad (27)$$

Substituting (27) into (26) yields:

$$\eta_{dis}(p_{ESS,dis}) = \frac{1}{1 + p_{ESS,dis} \left( \frac{R_{ESS} \cdot 10^3}{V_{dc}^2} \right)}. \quad (28)$$

Now, based on the efficiency losses, the charging power limits on the ESS can be derived by substituting (25) into (21):

$$\frac{\pi_{mean}}{\pi_{ch}} > \frac{1}{1 - p_{ESS,ch} \left( \frac{R_{ESS} \cdot 10^3}{V_{dc}^2} \right)} \quad (29)$$

$$\Rightarrow p_{max,ch|\eta} = \left( 1 - \frac{\pi_{ch}}{\pi_{mean}} \right) \left( \frac{V_{dc}^2}{R_{ESS}} \right) \cdot 10^{-3} \quad (30)$$

The inequality was changed to an equality sign in (30) since this is treated as the upper charging limit.

Similarly, the discharging power limits on the ESS can be derived by substituting (28) into (22):

$$\frac{\pi_{dis}}{\pi_{mean}} > 1 + p_{ESS,dis} \left( \frac{R_{ESS} \cdot 10^3}{V_{dc}^2} \right) \quad (31)$$

$$\Rightarrow p_{max,dis|\eta} = \left( \frac{\pi_{dis}}{\pi_{mean}} - 1 \right) \left( \frac{V_{dc}^2}{R_{ESS}} \right) \cdot 10^{-3} \quad (32)$$

Equations (30) and (32) show limits of the ESS' charging and discharging power in order to gain a benefit from arbitrage. In other words, too much energy would be lost due to inefficiencies that the arbitrage benefit would be lost at a power greater than imposed by these limits.

### C. Dispatch limit based on capacity

The storage capacity and remaining stored energy also introduce a limit on the ESS' power dispatch. With the given charging/discharging strategy, a limited capacity imposes a restriction on the ESS' ability to fully charge/discharge when the price is below/above the mean price. Looking at the price of energy in Fig. 2a and Fig. 2b, the ESS' available energy or capacity should be saved and utilized when the price difference is largest in order to maximize the benefit from arbitrage.

To obtain a maximal gain from arbitrage, the ESS should attempt to fully charge/discharge for every price area above/below a certain value; for demonstration purposes, the three areas marked "A" and "B" in Fig. 1 are examples of price areas that are above/below the mean value. This poses a problem since the actual size, number, and shapes of the future price areas are unknown without forecasting; however, the volatility of the price structure is presumed based on the price profile in the previous time periods.

The purpose of this limit is to ensure that there is sufficient capacity to charge/discharge the ESS when the price differences from the mean are largest. The strategy to determine the minimum power is described through the following steps<sup>2</sup>:

- 1) For every price area that is above  $\pi_{mean}$ ,
  - a) Sort the prices in decreasing  $\Delta\pi_{dis}$  price order.
  - b) Determine the amount of energy that the ESS discharges for each price based on the discharging strategy from Sections II-A and II-B.
  - c) Going from highest price differential to lowest, sum the energies until it equals the energy capacity of the ESS. The lowest price after which the ESS is depleted is the minimum price for that area. If the capacity is never reached, the minimum price difference is 0 \$/kWh.
  - d) Based on this price difference, determine the power of the ESS  $p_{min,dis|E,i}$  based on (13).
- 2) Determine  $p_{min,dis|E} = \min_i p_{min,dis|E,i}$  from all the price areas within the past  $T$ .

$p_{min,dis|E}$  is the minimum discharging power based on the ESS' capacity. Similarly,  $p_{min,ch|E}$  is the minimum charging power, and is found through a similar algorithm.

### D. Final Dispatch

Formally, the dispatch of the ESS is given below based on three different cases (charging, discharging, and idle). It is important to note that the application of these functions are mutually exclusive, which is evident as the ESS cannot both charge and discharge at the same time.

1) If  $\pi(t_0) < \pi_{mean}$ : then  $\Delta\pi_{ch}(t_0) = \pi_{mean} - \pi(t_0)$  and the ESS should charge. The power is based on (12):

$$p_{ESS}(t_0) = -p_{ESS,ch|P}(\Delta\pi_{T,ch}, t_0). \quad (33)$$

<sup>2</sup>Note: this example demonstrates the case when the ESS should be discharged; however, a similar algorithm can be applied to the case when it should be charging.

Note that the convention that the ESS is a generator is used, thus a negative power means that it is charging. The minimum power limit of the ESS in this case is:

$$p_{ESS,min}(t_0) = -\max\{0, p_{min,ch|E}\}, \quad (34)$$

where  $p_{min,ch|E}$  is evaluated from the algorithm derived in Section II-C. The maximum power limit of the ESS in this case is:

$$p_{ESS,max}(t_0) = -\min\{P_{ESS}, p_{max,ch|\eta}, p_{max,ch|C}\}, \quad (35)$$

where  $p_{max,ch|\eta}$  is calculated from (21), and  $p_{max,ch|C}$  is the charging limit based on remaining available capacity:

$$p_{max,ch|C}(t_0) = \frac{E_{ESS} - e_{ESS}(t_0)}{\Delta t} - p_{loss,ch} \quad (36)$$

Substituting (24) into (36) and solving for  $p_{max,ch|C}$  yields:

$$p_{max,ch|C}(t_0) = \frac{-\Delta t + \sqrt{\Delta t^2 - \frac{4R_{ESS}}{V_{dc}^2 \cdot 10^{-3}} \left( \frac{e_{ESS}(t_0) - E_{ESS}}{\Delta t} \right)}}{\frac{2R_{ESS}}{V_{dc}^2 \times 10^{-3}}}. \quad (37)$$

2) If  $\pi(t_0) > \pi_{mean}$ : then  $\Delta\pi_{ch}(t_0) = \pi(t_0) - \pi_{mean}$  and the ESS should discharge. The dispatched power is based on (13):

$$p_{ESS}(t_0) = p_{ESS,dis|P}(\Delta\pi_{T,dis}, t_0). \quad (38)$$

The minimum power limit of the ESS in this case is:

$$p_{ESS,min}(t_0) = \max\{0, p_{min,dis|E}\}, \quad (39)$$

where  $p_{min,dis|E}$  is evaluated from the algorithm derived in Section II-C. The maximum power limit of the ESS in this case is:

$$p_{ESS,max}(t_0) = \min\{P_{ESS}, p_{max,dis|\eta}, p_{max,dis|C}\}, \quad (40)$$

where  $p_{max,dis|\eta}$  is calculated from (22), and  $p_{max,dis|C}$  is the charging limit based on remaining available capacity:

$$p_{max,dis|C}(t_0) = \frac{e_{ESS}(t_0)}{\Delta t} - p_{loss,dis} \quad (41)$$

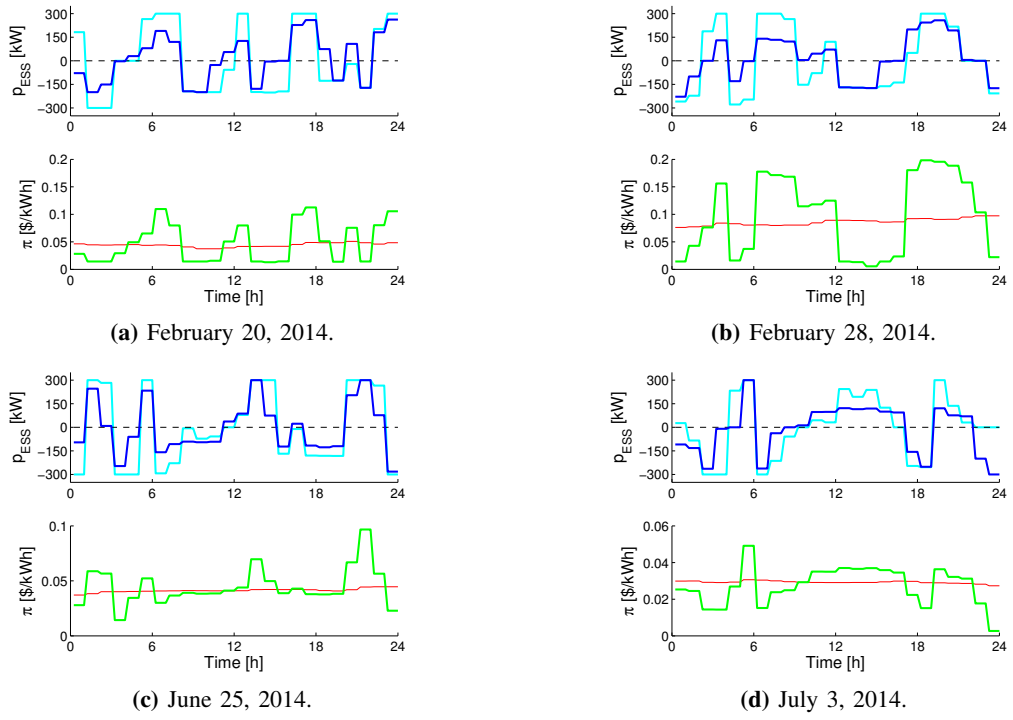
Substituting (27) into (41) and solving for  $p_{max,dis|C}$  yields:

$$p_{max,dis|C}(t_0) = \frac{-\Delta t + \sqrt{\Delta t^2 - \frac{4R_{ESS}}{V_{dc}^2 \cdot 10^{-3}} \left( \frac{-e_{ESS}(t_0)}{\Delta t} \right)}}{\frac{2R_{ESS}}{V_{dc}^2 \times 10^{-3}}}. \quad (42)$$

3) If  $\pi(t_0) = \pi_{mean}$ : then  $p_{ESS}(t_0) = 0$  kW.

Finally, the power limit of the transformer at the point of common coupling introduces a power limit on the ESS such that:

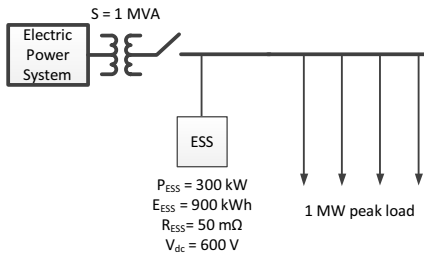
$$|p_{ESS}(t) - p_{load}(t)| \leq |P_{EPS}|. \quad (43)$$



**Fig. 4:** Example daily dispatch profiles of the ESS with the backcasting NSEV algorithm (blue) as compared to the ideal forecasting dispatch (cyan). Below each dispatch figure shows the market price of energy (green) with the calculated mean price of energy from the past 24 hours (red).

### III. SYSTEM DESCRIPTION

The backcasting NSEV algorithm is employed in the system depicted in Fig. 5. The peak load in the system is 1 MW, and is based on the load profile in 2014 taken from the Independent Electricity System Operator (IESO) in Ontario [31]. The market price of electricity is also taken from the IESO in 2014 [31], upon which the ESS will participate to perform its arbitrage. In this study, the microgrid's operation has negligible market influence due to its small size relative to the Electric Power System (EPS).



**Fig. 5:** Single Line Diagram of test system.

Two base cases are used in the paper to act as benchmarks against which the proposed NSEV algorithm is compared:

- 1) one without any storage or ED algorithm, which represents the worst case scenario for reducing the price of energy, and
- 2) one with a deterministic optimization implemented in GAMS with ideal 3-day forecast values, which repre-

sents the best case scenario for reducing the price of energy.

It is assumed that the price of energy from the EPS is unaffected by the amount of power import or export from the microgrid; that is, the microgrid is sufficiently small to not play a large role on the market prices [32]. Only results from the grid-connected mode of operation are shown.

### IV. RESULTS

Fig. 4 shows four examples of the dispatch profile for separate days, and how the dispatch reacts to the changing market prices of energy as compared to the case with ideal forecasting. There is a strong correlation between the ideal and proposed dispatches, where the ESS typically charges and discharges. Over the year, the price of energy is reduced by 8.14% as compared to the base case without storage.

A parametric analysis of the ESS' power rating (Fig. 6), capacity (Fig. 7), and internal resistance (Fig. 8) shows how the algorithm adapts to the different ESS parameters. The simulations are run over an entire year and the overall average price of energy of the microgrid is calculated and plotted versus the respective parameters. This demonstrates the annual net profit obtained through the NSEV algorithm distributed over the total energy cost in the microgrid. Overall, the proposed backtracking algorithm is able to take advantage of the extra power, energy, and efficiency. The NSEV algorithm achieves a respective mean reduction in the average cost of energy to the microgrid by 28.1% (Fig. 6), 30.2% (Fig. 7), and 27.7% (Fig. 8) as compared to the base case, and a reduction

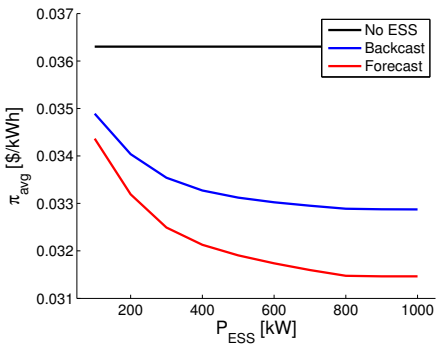


Fig. 6: Average annual price versus the ESS' power rating.

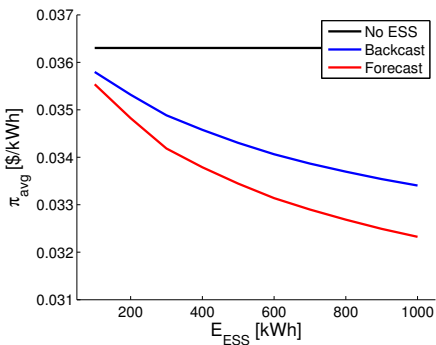


Fig. 7: Average annual price versus the ESS' energy rating.

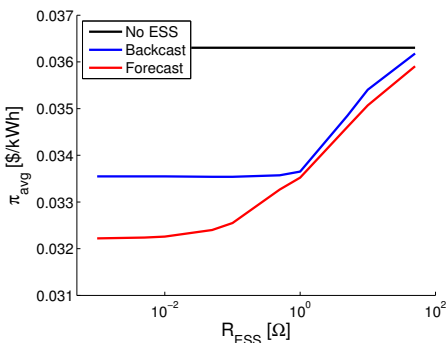


Fig. 8: Average annual price versus the ESS' internal resistance.

of the average cost of energy by up to 72.3% of the ideal case that was determined through a perfect forecasting dispatch.

### V. DISCUSSION

The proposed backcasting algorithm has been shown to reduce the average price of energy to the microgrid consumers simply through arbitrage. This algorithm is best implemented when forecasting algorithms are either erroneous or difficult to implement. The main drawback of the algorithm is that it is impossible to foresee any minor fluctuation on the price of energy of which it can take advantage (such as  $t = [6, 12]$  h from Fig 4b), which a dispatch algorithm with full foresight is able to modify its charging/discharging pattern. In reality, however, classic forecasting algorithms are also prone to errors and may not capture this opportunity because the given price pattern exhibited is essentially a statistical outlier.

The backcasting algorithm continuously adapts to the system to which it is connected by recalculating the NSEV at each time step. The NSEV algorithm can be used to determine the precise charging/discharging power of the TDR based on the marginal price of energy when the microgrid is grid-connected for arbitrage purposes, or it can seamlessly be integrated in the economic dispatch algorithm when the microgrid is operating in islanded mode. When islanded, the amount of power charging/discharging the ESS will affect power operation points of other DERs, and thus will affect the marginal price of energy. By introducing the linear marginal price curve, the ESS' valuation function can be considered similar to a thermal resource in a classical economic dispatch.

### VI. CONCLUSION

The focus and scope of this paper is to determine the net stored energy value in a time-dependent resource as a function of the energy price as compared to past time frames. The proposed backcasting algorithm considers operating limits such as variable efficiency, power rating, stored energy, and energy rating, which none of the other algorithms in the literature address simultaneously. When using real market prices and load profiles, the results show that the average price of energy in the microgrid is reduced by 8.14%.

The applicability of this work is to microgrid systems that employ a form of time-dependent resource, such as energy storage. This formulation presents the time-dependent resource as if it were time-independent to the energy management system. Since the final marginal price curve is linear (as is the case with thermal-based generators), the proposed approach supplements the classical economic dispatch by facilitating the integration of energy storage systems into its optimization algorithm. The backcasting approach is best utilized when detailed or accurate forecasting algorithms are unavailable, or if the stochastic distribution of the uncertain variable is subject to change since the NSEV algorithm is adaptable to current operating characteristics. Future work involves incorporating this formulation into a microgrid controller so that it can be used in an economic dispatch in both grid-connected and islanded modes of operation with multiple distributed energy resources, including various generators and load types.

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