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New approach for strategic bidding of Gencos in energy and spinning reserve markets

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Abstract

In restructured and de-regulated power systems, generating companies (Gencos) are responsible for supplying electricity for both energy and reserve markets, which usually operate simultaneously. In this condition, the question is how much and for what price must each Genco generate for each market to maximize its profit, so this paper intends to answer to this question. In this paper, first, the combined energy and reserve markets are considered, and the Nash equilibrium points are determined. Then, the bidding strategies for each Genco at these points will be presented. The bids for the energy and 10 min spinning reserve (TMSR) markets are separated in the second stage, and again, the bidding strategies for each Genco for the two separated markets will be demonstrated. Comparison of the results shows that the separated bidding strategies, while being simplified with the algebraic optimization model and reducing the time consumed, give the same results as the combined ones. The Western System Coordinating Council (WSCC) nine bus test system is employed to illustrate and verify the results of the proposed method.

Keywords: Energy market; Gencos; Nash equilibrium point; Optimal bidding strategy; Reserve

1. Introduction

In the liberalized electricity market, which is the dominant structure of the power system around the world, the Gencos sell their generation to the energy market and try to maximize their profit. In this environment, an independent organization, normally called the PX or ISO (independent system operator), determines the purchasing cost based on the receiving bids of all the Gencos.

Gencos are very interested in selling their outputs at higher prices, but due to the competition, they may be eliminated from the market in some hours because of their bid being too high in price or amount. The profit for a Genco will be the difference between its accepted bidding selling price and its operational cost.

Gencos can sell their outputs to different markets: energy market or ancillary service markets (such as reserve market and reactive power market). So the Gencos must devise a good bidding strategy in these markets to obtain the maximum profit. Usually, the bidding strategies of Gencos are based on their opponents' bidding behavior, the forecasted demand, the required spinning reserve and, last but not least, the power system operating conditions.

There are two types of methods for developing bidding strategies in electricity markets: game based and non-game based methods. The game based method, which will be used in this article, utilizes game theory to simulate the bidding behaviors of Gencos and develop Nash equilibrium bidding strategies for them in electricity markets [1]. The game based method can be classified in the following categories [1,2]:

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(a) Complete and incomplete information.
(b) Cooperative and non-cooperative game.
(c) Perfect and imperfect information.

Incomplete information gaming can be transferred to a complete game with imperfect information [1]. Ref. [3] has used a complete information model and presented a methodology to design an optimal bidding strategy for a generator according to its degree of risk aversion, the forecasted price and the probability distributions of errors in the forecasted price for each hour of a day. With the forecasted price, profit maximization was performed to find the optimal value of production, and consequently, the bidding curve was obtained. Ref. [4] extended the proposed methods in Refs. [1,2] to establish a more general approach, which is appropriate for Gencos' optimal bidding strategies with incomplete information while taking into account transmission constraints.

In Ref. [5], the bidding problem was modeled as a bi-level problem by assuming complete information on a Genco's opponents. The ISO's market clearing problem was modeled as a non-linear optimal power flow problem, and a Newton approach was employed to solve it. A method to predict the optimal energy production of a power producer in an oligopoly energy market was presented in Ref. [6], but the model did not consider the technical constraints of the generation companies.

Ref. [7] described the PJM (Penn–Jersey–Maryland) spinning reserve market. In Ref. [7], a descriptive overview of the PJM spinning reserve market and a list of the PJM spinning reserve market rules can be found. In Ref. [8], two general forms of reserve market are introduced. These two forms depend on whether reserve payments are made for actual power delivered or for power that is merely reserved.

This paper presents a method for developing the Genco's optimal bidding strategies in energy and TMSR markets. In this paper, the bidding strategy problem is formulated as a bi-level problem in which the upper level sub-problem maximizes the Genco's payoffs, and the lower level sub-problem solves the ISO's market clearing problem. The algorithm used to solve the ISO's objective function is based on the Lagrangian method.

The main contributions of this paper are as follows: (i) Develop optimal bidding strategies for Gencos in energy and TMSR markets and not only the energy market as most research on the subject focuses, (ii) Complete techno-economical modeling of Gencos and transmission systems.

This paper is organized as follows. The basic concepts of game theory are given in Section 2. Section 3 describes the problem formulation, and the proposed solution methods are debated in Section 4. Section 5 gives an illustrative example with three Gencos, and Section 6 provides the conclusion.

2. Basic concepts of game theory

Generally, the strategic behavior of generating companies falls into the category of game theory problems. Therefore, it is necessary to review the basic concepts of game theory and Nash equilibrium definition.

Game theory is the study of multi-person or multi-firm decision making problems. In the field of industrial organization in economics, game theory is used extensively to study auction behavior, bargaining, principal–agent relationships, product differentiation and strategic behavior by firms. There are three main mathematical models or forms used in the study of games, the strategic form, the extensive form and the coalitional form [9]. These approaches differ in the amount of detail on the play of the game built into the model. The history of electricity markets shows that their behavior is near to the strategic form [10]. Therefore, this section explains the basic concepts of this form.

The strategic (or normal) form representation of a game includes three components [11]:

- The set of players, $i \in \{1, \ldots, n\}$, in the game, which is assumed finite.
- The pure strategy space, S_i, which contains the individual strategies available to player i (s_{ij}), where s_{ij} is an arbitrary strategy and
- the payoff function $u_i : S \to R(\mathit{real\ set})$ for each player i is also defined, where $S = S_1 \times S_2 \times \cdots \times S_n$ is the Cartesian set of all sets (S_i).

In game theory, the most commonly encountered solution concept is Nash equilibrium. A strategy is a Nash equilibrium strategy for a player if that player will decrease its payoff if it deviates from its Nash equilibrium strategy, assuming all other players continue to play their existing strategies. As a result, a Nash equilibrium point is a "best response", in the sense that no player has an incentive to deviate from its strategy choice, given all other player's strategy choices. Definition 1 gives a formal definition of Nash equilibrium.

Definition 1. In the n player strategic form game, the profile strategies $(s_1^*, \ldots, s_n^*)$ are a Nash equilibrium if, for each player i, $s_i^* \in S_i$ is player i's best response to the strategies specified for the other $(n-1)$ players (its opponents), $s_{-i}^* = (s_1^*, \ldots, s_{i-1}^*, s_{i+1}^*, \ldots, s_n^*)$, such that $u_i(s_i^*, s_{-i}^*) \geqslant u_i(s_{ij}, s_{-i}^*)$, for every feasible strategy $s_{ij} \in S_i$ [11].

3. Problem formulation

3.1. Estimating opponents unknown information

Generally, the Gencos do not have access to complete information of their opponents, so it is necessary for a Genco to model its opponents' unknown information to predict their behavior in the market.

If it is supposed that all Gencos own only thermal units, the most important parameters for Gencos will be the coefficients of the second order generating cost function as

$aP^2 + bP + c$ where P is the active power output of a generating unit.

The available information to Gencos about their opponents is incomplete, and it is supposed that they are only aware of the minimum and maximum generation levels of their opponents as well as their fuels type.

Ref. [12] has presented a method to obtain the fuel cost of a generator as a quadratic function of its active power generation. This function is expressed as

$$F(P) = \alpha P^2 + \beta P + \gamma \tag{1}$$

In this function, $F(P)$ is measured in MJ/h or MBtu/h, so considering the higher heating value (HHV) of fuels and the fuels price (in \$/m^3 or \$/l), $F(P)$ is obtained in \$. So the fuel price must be forecasted for future time to obtain the fuel cost. We can define several scenarios with definite probabilities for the fuel price, so the different types for the α, β and γ coefficients will result.

The total cost of operation includes the fuel cost, the cost of labor, supplies and maintenance. These costs, except the fuel cost, are expressed as a fixed percentage of the fuel cost. So the total generation cost can be expressed by $aP^2 + bP + c$ where a, b and c include α, β and γ plus some percentage due to the cost of labor, maintenance and supplies.

3.2. Genco's bids in energy market

In a power market, Gencos may prepare their strategic bids according to the four known models in imperfect competition, i.e. some firms (the strategic players) are able to influence the market price through their actions. These models are the Bertrand, Cournot, Stackelberg and supply function equilibrium (SFE) models [11].

In the Bertrand model, Gencos compete with each other using prices as strategy choices, and in perfect competition, they bid at their marginal cost at their Nash equilibrium point.

In the classic model of Cournot, Gencos compete against each other using quantities as strategy choices. In this model, the Genco's products are assumed to be homogeneous; demand is price responsive; and the market clearing price (MCP) is the intersection of the aggregated supply and market demand curves. The Stackelberg model is similar to the Cournot model. However, the competitors do not offer their output quantities simultaneously. The so called "leader" will make the first move, which is followed by that of followers who take into account the leader's action [10].

In the SFE model, Gencos compete with each other through the simultaneous choice of supply functions. Klemperer and Meyer developed the SFE model in order to model competition in the presence of demand uncertainty. The SFE model was used by Green and Newbery for analyzing the competitive strategic bidding in electricity markets [11].

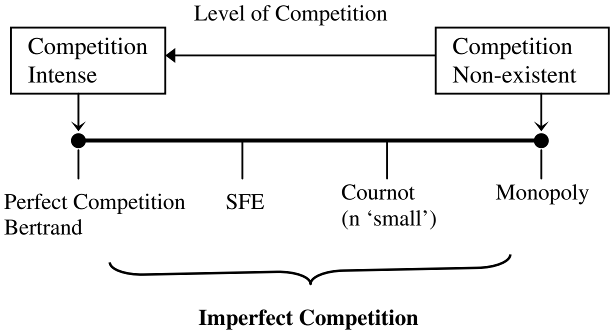

Fig. 1. Equilibrium models and predicted degree of competition [9].

Fig. 1 illustrates where the intensity of competition predicted by the basic formulation of each of the models places them along the competitive spectrum.

Among these models, it is only the SFE model that enables a Genco to link its bidding price with the bidding quantity of its product, and only this model is the closest to the actual behavior of players in the actual power market.

Let us suppose that the Gencos are requested to submit a piece wise quantity-price curve like the one shown in Fig. 2 for each generator to the ISO for the energy market. Accordingly, for the jth generator, the Gencos would devise their own bid segments according to the linear supply function [4]:

$$\rho_{ji} = k_j \cdot \mathrm{MC}_{ji} = k_j \cdot (2a_j P_{ji} + b_j) \tag{2}$$

where i is bid block; ρ_{ji} is the bidding price for the ith block of generator j; k_j is the bidding strategy of generator j; MC_{ji} is the marginal cost for the ith block of generator j with second order generation cost function as $C_{ji} = C(P_{ji}) = a_j P_{ji}^2 + b_j P_{ji} + c_j$; and a_j, b_j and c_j are generation cost coefficients.

The bid pairs submitted to the markets are

$$(P_{j1} - P_{\min}, \rho_{j1}), (P_{j2} - P_{j1}, \rho_{j2}), \quad \text{and} \quad (P_{j3} - P_{j2}, \rho_{j3})$$

In the multi-block case, the Gencos have to deal with more decision variables such as the number of blocks, a bidding strategy for each block and the amount of power for each block. To simplify the problem, it is assumed that the number of blocks for each Genco is equal to three. So each Genco deals with four independent variables. These variables are: $k_j, P_{j1}, P_{j2}, P_{j3}$.

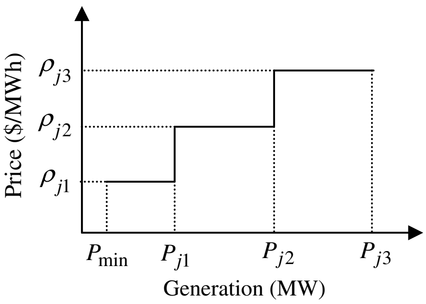

Fig. 2. Gencos bid curve for generator j in the energy market.

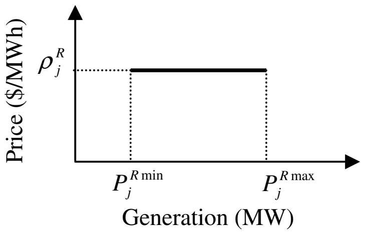

Fig. 3. Gencos bid curve for generator j in the reserve market.

Naturally, the amount of P_{j3} should be equal to the total capacity of the jth generator, but some Gencos may withhold part of their whole capacity to sell the remaining capacity in the other markets, such as the TMSR market. Therefore, it is critical for a Genco to make a good decision about sharing its total capacity among the energy and other markets. In this paper, it is supposed that a Genco's bid curve in TMSR market is a single block case as shown in Fig. 3.

In this figure, ρ_j^R, $P_j^{R\min}$ and $P_j^{R\max}$ are the bidding price, minimum and maximum offered quantities, respectively, for generator j in the reserve market.

3.3. Market clearing model

In the restructured power systems, the Gencos will submit bid curves to the ISO. Then, the ISO clears the market after collecting all bids. In the ISO's market clearing model, the ISO dispatches generating units in order of lowest to highest bid as needed to meet demand with considering network constraints.

Once the energy market is cleared, each generating unit will be paid according to the pricing mechanism of the market. Generally, there are two pricing mechanisms: uniform and pay as bid [13].

Definition 2 (*Uniform clearing price* (*UCP*)). Under the uniform pricing structure, the bid price of the last unit dispatched sets the market clearing price, and then, all units dispatched receive the same market clearing price (MCP).

Definition 3 (*Pay as bid pricing*). Under the pay as bid pricing structure, every winning generating unit gets its bid price as its income.

Therefore, a Genco's payoff will be different based on the pricing mechanism, which affects the bidding strategies of the Gencos. In this paper, it is supposed that the market structure is based on uniform pricing. Therefore, all winning Gencos get the MCP as their income.

3.4. General forms of a reserve market

In competitive electricity markets, Gencos sell electricity in the energy market and sell spinning and non-spinning reserve in the reserve markets. The reserve payment mechanism can be made in two scenarios: payment for power delivered and payment for reserve allocated [8].

In the first scenario, a generator, which sells a portion of its power as reserve is paid the reserve price only if the reserve power is actually used. The reserve price is usually higher than the energy price. In this case, a generator receives a profit on sales of reserve only for the time periods when the reserve actually needs to be generated. The generator receives zero payment if the reserve is not called.

Therefore, the obtained profit of generator j in the energy and TMSR markets will be expressed as follows:

$$\begin{aligned}\pi_j = {} & \mathrm{MCP}^{\mathrm{E}} * P_j^{\mathrm{awarded}} + r * \mathrm{MCP}^{\mathrm{R}} * P_{j\mathrm{R}}^{\mathrm{awarded}} \\ & - (1-r)\cdot(a_j * (P_j^{\mathrm{awarded}})^2 + b_j * P_j^{\mathrm{awarded}} + c_j) \\ & - r(a_j(P_j^{\mathrm{awarded}} + P_{j\mathrm{R}}^{\mathrm{awarded}})^2 + b_j(P_j^{\mathrm{awarded}} + P_{j\mathrm{R}}^{\mathrm{awarded}}) + c_j)\end{aligned} \tag{3}$$

where π_j is the obtained profit in the energy and TMSR markets; $\mathrm{MCP}^{\mathrm{E}}$ and $\mathrm{MCP}^{\mathrm{R}}$ are the market clearing price (MCP) of the energy and reserve markets, respectively; P_j^{awarded}, $P_{j\mathrm{R}}^{\mathrm{awarded}}$ are the awarded generation of generator j in the energy and reserve markets, respectively; r is the probability of calling reserve power; and a_j, b_j, c_j are the coefficients of the generating cost function.

In the second scenario, a generator receives the price per unit of reserve power during the time period that the reserve is allocated and not used. If the reserve is used, then the generator receives the spot price for the reserve power that is generated. In this case, the price of reserve will be much lower than the spot price of power, but it should be high enough so that the generator expects an overall long term profit; otherwise, no reserve would be offered for sale [8].

Therefore, the obtained profit of generator j in the energy and TMSR markets in this case will be expressed as follows:

$$\begin{aligned}\pi_j = {} & \mathrm{MCP}^{\mathrm{E}} * P_j^{\mathrm{awarded}} + (1-r) * \mathrm{MCP}^{\mathrm{R}} * P_{j\mathrm{R}}^{\mathrm{awarded}} \\ & + r * \mathrm{MCP}^{\mathrm{E}} * P_{j\mathrm{R}}^{\mathrm{awarded}} - (1-r) * (a_j * (P_j^{\mathrm{awarded}})^2 \\ & + b_j * P_j^{\mathrm{awarded}} + c_j) - r * (a_j(P_j^{\mathrm{awarded}} + P_{j\mathrm{R}}^{\mathrm{awarded}})^2 \\ & + b_j * (P_j^{\mathrm{awarded}} + P_{j\mathrm{R}}^{\mathrm{awarded}}) + c_j)\end{aligned} \tag{4}$$

where the parameters are the same as described in Eq. (3).

4. Problem solution

As the energy and TMSR markets operate simultaneously, this section proposes two methods for developing bidding strategies of the Gencos in two markets:

A: Combined energy and reserve markets.
B: Separate energy and reserve markets.

4.1. Combined energy and reserve markets

As previously mentioned, the energy and TMSR markets usually operate simultaneously. Therefore, Gencos should choose the best bidding strategies in these two markets to maximize their profits. In order to reach this target, they should consider their opponent's activities and the power system conditions. Hence, each Genco, in order to choose the best bidding strategies should solve a bi-level problem, in which the upper level sub-problem maximizes the individual Genco's payoffs and the lower level sub-problem solves the ISO's market clearing problem after modeling the opponent's behaviors. Each Genco has knowledge of its own payoffs and generation costs but could lack such information on the other Gencos. Hence, they should model their opponents with approximate information as described in Section 3.1. In the lower level sub-problem, the bidding strategies of all Gencos with regard to network conditions are analyzed to obtain the dispatched quantity of each Genco. In this level, where the reserve payments are for power delivered, the ISO solves the following co-optimization problem for two markets:

$$\begin{aligned} \min \quad & \sum_{j=1}^{N}\sum_{b=1}^{B_j}\rho_{jb} * P_{jb} + \sum_{j=1}^{N}\rho_j^R * P_j^R \\ \text{s.t.} \quad & Y\theta = P_G - P_D \\ & F_l^{\min} \leqslant F_l \leqslant F_l^{\max} \\ & P_{jb}^{\min} \leqslant P_{jb} \leqslant P_{jb}^{\max} \\ & P_j^{R\min} \leqslant P_j^R \leqslant P_j^{R\max} \\ & P_j^R \leqslant 10R_{\mathrm{amp}_j} \\ & \sum_{j=1}^{N} P_j^R = SR \\ & \rho_j^R \leqslant \rho_{\max} \end{aligned} \tag{5}$$

where N is the number of generators; B_j is the number of blocks for generator j; ρ_{jb} is the offered price and quantity for bth block of generator j; P_{jb} is the awarded quantity for the bth block of generator j; ρ_j^R is the bidding price and quantity of generator j in the TMSR market; P_j^R is the awarded quantity of generator j in the TMSR market; Y is the network admittance matrix; θ is the vector of bus voltage angles; P_G is the vector of active power bus generation; P_D is a constant vector of active power bus loads; $F_l^{\min}$, $F_l^{\max}$ are the lower and upper real power flow limits on line l, respectively; F_l is the power flow on line l; $P_{jb}^{\min}$, $P_{jb}^{\max}$ are the lower and upper bounds of block b for generator j, respectively; $P_j^{R\min}$, $P_j^{R\max}$ are the lower and upper levels of reserve quantity, respectively; R_{amp_j} is the ramp rate of generator j (MW/min); SR is the required amount of TMSR; and $\rho_{\max}$ is the cap price of the reserve market.

In Eq. (5), the first equality constraint is the DC power flow equation, the second constraint is the transmission line constraint, the third and fourth constraints are the generation capacity constraints in the energy and TMSR markets and the last three equalities are the reserve market constraints.

Besides the MWs dispatched by the generators, the energy and reserve market clearing prices are the other results of this optimization problem. Each generator will receive the market clearing price times its awarded generation in each market.

After the market clearing stage, the Gencos have knowledge about their obtained profit, and they change their strategies until their strategies converge to the Nash equilibrium point. In this step, the Gencos decide on their offers considering the results of the last iteration as a way of taking into account the behavior of other opponents. This stage is composed of two different parts: Modification of price bidding strategies and modification of the amount of each power block.

There are two methods for updating a generator's bidding strategies [4], where, in this paper, the following method is used:

(a) Set the initial values of k_j, P_{j1}, P_{j2} and P_{j3} for the energy market and ρ_j^R, $P_j^{R\min}$ and $P_j^{R\max}$ for the reserve market for each generator.
(b) Suppose the bidding strategies of the opponents' generators are fixed and update Genco i's bidding strategies for its units until no unit will change its bidding strategy.
(c) Repeat (b) to find each Genco's optimal bidding strategies in response to their opponents' bidding strategies.
(d) Go to (b) and repeat the procedure until no generator would change its bidding strategy.

This method will converge to the Nash equilibrium point, which is the solution of the problem and is shown in Fig. 4.

4.2. Separate energy and reserve markets

As previously mentioned, when Gencos want to maximize their total payoffs in two markets, they should choose the best strategies for seven apparently independent variables, namely k_j, P_{j1}, P_{j2}, P_{j3}, ρ_j^R, $P_j^{R\min}$ and $P_j^{R\max}$. It is obvious that the large number of variables will lead to a complicated problem, and due to the non-linearity of the problem, the solution of the optimization problem may not be the global solution. If the number of independent variables is reduced, then it can be possible to simplify the problem and speed up convergence of the algorithm. Therefore, this section proposes a new method for solving this complexity.

Usually, an ISO buys the required TMSR from the dispatched Gencos in the energy market. Therefore, Gencos should try to win in the energy market to have the chance to participate in the reserve market. On the other hand, since the reserve is not called most of the time (hopefully!), the obtained profit in the energy market is much more observable and important than that in the reserve market [8]. With regard to these conditions, the following method is proposed to find the optimal bidding strategies of Gencos in two markets:

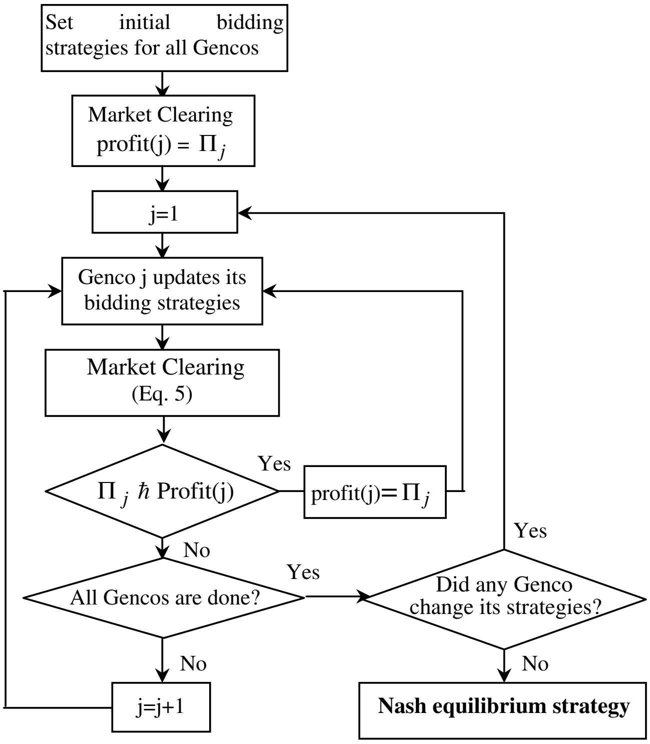

Fig. 4. Solution of combined markets.

1. Find the Nash equilibrium point for the bidding strategies of Gencos in the energy market without considering the reserve market.
2. Allocate the remaining capacity of the Gencos for bidding in the reserve market.
3. Find the Nash equilibrium bidding strategies of the Gencos in the TMSR market.

The remaining capacity, if any, can be used for the other ancillary service markets such as the reactive power market.

The solution method, which is used for iterations 1 and 3 is the same as the algorithm shown in Fig. 4, but the market clearing model is different from Eq. (5). Based on the proposed method, appropriate models for market clearing in iterations 1 and 3 are expressed in Eqs. (6) and (7), respectively.

$$\begin{aligned}
\min \quad & \sum_{j=1}^{N} \sum_{b=1}^{B_j} \rho_{jb} P_{jb} \\
\text{s.t.} \quad & Y\theta = P_G - P_D \\
& F_l^{\min} \leqslant F_l \leqslant F_l^{\max} \\
& P_{jb}^{\min} \leqslant P_{jb} \leqslant C_j
\end{aligned} \tag{6}$$

$$\begin{aligned}
\min \quad & \sum_{j=1}^{N} \rho_j^R \cdot P_j^R \\
\text{s.t.} \quad & P_j^{R\min} \leqslant P_j^R \leqslant P_j^{R\max} \\
& P_j^R \leqslant 10 R_{\mathrm{amp}_j} \\
& \sum_{j=1}^{N} P_j^R = SR \\
& \rho_j^R \leqslant \rho_{\max}
\end{aligned} \tag{7}$$

In Eq. (6), C_j is the nominal capacity of generator j, and in Eq. (7), $P_j^{R\max}$ is the difference between the nominal capacity of generator j and its awarded quantity in the energy market. The other parameters are the same as those in Eq. (5).

These game problems may have only one Nash equilibrium point, multiple Nash equilibria. The upper limit considered for the bidding strategy (such as k) of the Gencos may also be an affecting factor in multiple Nash equilibria. The computational requirement for the proposed algorithm will increase with the number of Gencos. Meanwhile, a Genco may speed up the convergence of the algorithm by providing a good estimate for the initial bidding strategy.

5. Numerical example

The WSCC 9 bus system (Fig. 5) is used to illustrate the implementation of the proposed method for a typical power system. There are three units, and each unit is supposed as a Genco. The information on the load service entities (LSEs) and the network is shown in Tables 1 and 2, respectively. It is supposed that the Gencos can predict the exact fuel price, hence, one generation cost structure is defined in Table 3.

In the following case studies, we suppose that the bidding strategy of the Gencos varies between 1 and 2.5 times of its marginal cost to exhibit how expected payoff and reserve payment values are affected by the strategies adopted by the Gencos for participation in two markets. Also, it is assumed that the reserve payment mechanism

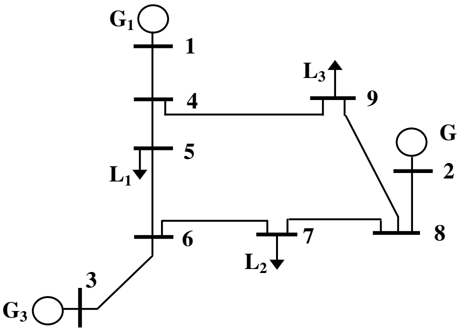

Fig. 5. WSCC nine bus system.

Table 1
LSEs' information

LSE	L1	L2	L3
MW	175	180	200

Table 2
Transmission network data

Line	From bus	To bus	R (pu)	X (Pu)	Limit (MW)
1	1	4	0	0.0576	250
2	4	5	0.017	0.092	250
3	5	6	0.039	0.17	150
4	3	6	0	0.0586	300
5	6	7	0.0119	0.1008	150
6	7	8	0.0085	0.072	250
7	8	2	0	0.0625	250
8	8	9	0.032	0.161	250
9	9	4	0.01	0.085	250

Table 3
Cost coefficients of Gencos

Genco	a	b	c	$P_{\min}$ (MW)	$P_{\max}$ (MW)	$\mathrm{Ir}_{\mathrm{amp}}$ (MW/min)
1	0.11	5	150	10	250	6
2	0.085	1.2	600	10	300	6
3	0.1225	1	335	10	270	6

is based on the payment for power delivered. All case studies are calculated considering 110 ($/MW) for the TMSR market cap price. The required amount of TMSR is equal to 20% of the total demand with the probability of 0.005 for called and generated reserve power.

Case 1 In this case, Gencos bid at their marginal cost in the energy market, and their price bidding in the TMSR market is equal to the cap price. The ISO clears the energy and TMSR markets using unconstrained security unconstrained (transmission lines have no flow limit) and constrained economic dispatch Eq. (5). Tables 4 and 5 show the expected payoff, generation dispatch and TMSR dispatch of the Gencos. This case will be used to compare other cases.

Table 4
MW dispatched and total payoff in the energy and TMSR markets without network constraints

	Genco1	Genco2	Genco3
Expected payoff ($)	463.1	11,094	8205.9
MW dispatched in energy market	10	300	263.8
MW dispatched in TMSR market	37	37	37

Table 5
MW dispatched and total payoff in the energy and TMSR markets with network constraints

	Genco1	Genco2	Genco3
Expected payoff ($)	4340.1	5722.1	4540.3
MW dispatched in energy market	172	200	196
MW dispatched in TMSR market	37	37	37

Table 6
Marginal cost of Gencos correspond to the awarded energy and the energy market clearing price

	Price bidding ($/MW h)			MCP ($/MW h)
	Genco1	Genco2	Genco3	
Unconstrained network model	7.2	52.2	65.6	65.6
Constrained network model	42.84	35.2	48.98	48.98

The marginal cost of the Gencos corresponding to their awarded generation and the MCPs are shown in Table 6 for the unconstrained and constrained network models. As shown in these tables, Genco 3 is the marginal unit (which sets the MCP).

Because of the higher amounts of the a and b coefficients of Genco 1 than the others, Genco 1 has a small share in this market.

Case 2 In this case, the Gencos use the method introduced in Section 4.2 for calculating their optimal bidding strategies in the energy and TMSR markets. The optimal bidding strategy, the optimal amount of power for the blocks and the MWs dispatched in the energy market are shown in Tables 7 and 8.

Based on the dispatched MWs in the energy market, the offered quantities for the TMSR market is determined, and the Gencos use the described model in Section 4.2 to produce their optimal bidding strategy in the TMSR market. The offered quantities, the awarded quantities and the optimal bidding strategies for the TMSR markets are shown in Tables 9 and 10. With regard to the knowledge of all Gencos about the cap price of TMSR market, it is reasonable that they bid at the cap price.

The expected pay off of the Gencos is shown in Table 11 for the unconstrained and constrained network models.

Table 7
Optimal bidding strategies, optimal amount of power for blocks and the MWs dispatched in the energy market without network constraints

	Bidding strategy (k)	P_1	P_2	P_3	MW dispatched
Genco1	1.1628	80	160	250	175
Genco2	1.02	100	200	–	200
Genco3	1	100	180	270	180

Table 8
Optimal bidding strategies, optimal amount of power for blocks and the MWs dispatched in the energy market with network constraints

	Bidding strategy (k)	P_1	P_2	P_3	MW dispatched
Genco1	1	80	160	–	160
Genco2	1.04	100	220	300	220
Genco3	1	100	180	270	183.2

Table 9
The offered and awarded quantity and the optimal bidding strategies in the tmsr market without network constraints

	Offered quantity (MW)	Awarded quantity (MW)	Price bidding ($)
Genco1	75	37	Cap price
Genco2	100	37	Cap price
Genco3	90	37	Cap price

Table 10
The offered and awarded quantity and the optimal bidding strategies with network constraints

	Offered quantity (MW)	Awarded quantity (MW)	Price bidding ($)
Genco1	90	37	Cap price
Genco2	80	37	Cap price
Genco3	86.8	37	Cap price

Table 11
The total expected pay off of Gencos in two markets

	Genco1	Genco2	Genco3
Unconstrained network model	7833.5	12,230	8374.2
Constrained network model	7640	10,697.9	8410.6

As shown in Tables 9 and 10, the awarded quantities of the Gencos in the TMSR market are adjusted with the ramp rate constraint of the Gencos. The comparison between the results of this case and the ones in case 1 (see Tables 4, 5 and 11) shows that it is profitable for the Gencos to bid strategically rather than to bid at their marginal cost.

Case 3 In this case, we perform a sensitivity analysis on the probability of calling reserve. As previously mentioned, the low amount of r (the probability that the reserve power is called and generated) was an important factor for optimizing the strategies of Gencos in the reserve market after determining their optimal strategies in the energy market (the proposed method). So it is obvious that when r is near 1.0, it may be profitable for Gencos to change their strategies from the Nash equilibrium in the energy market and allocate a larger amount of quantity for the reserve market. The results of case 2 show that because of enough remaining capacity from the Nash equilibrium point for all the Gencos to sell power in the TMSR market (Tables 9 and 10), it is not profitable for them to change their strategies. So, in order to show this sensitivity analysis, the LSEs are increased to 1.1 times the previous amounts. In this condition, when a Genco withholds its capacity, the share amount of their opponents in the energy market is increased, and then, its share in providing reserve will be increased and more benefit may be obtained when the calling probability increases. Therefore, in this case, the Gencos change the strategies of the seven apparently independent variables simultaneously and use the model described in Section 4.1 to obtain the Nash equilibrium points.

In this case, the probability r is varied between 0.05 and 1. Table 12 shows the optimal allocated quantity (P_3) for the energy market and the awarded quantity in the TMSR market considering the network constraints. Each cell in this table shows these quantities for Gencos 1, 2 and 3. For example, the values of the third row and second column of Table 12 are 160, 300 and 270. These values are the optimal allocated quantity for Gencos 1, 2 and 3, respectively, when r is equal to 0.1. As shown in this table, when r is lower than 0.3, the optimal allocated quantity for the two markets is identical to the corresponding amount of Table 8, and when r is greater than 0.3, the optimal energy quantity bidding is different from the Nash equilibrium quantity in

Table 12
Optimal quantity for energy and TMSR markets considering network constraints

r	Optimal P_3 (MW)	Awarded quantity in TMSR market (MW)	Expected pay off ($)
0.05	160	41.2	7822.1
	300	41.2	10,868.3
	270	28.6	9601.8
0.1	160	41.2	7822.1
	300	41.2	10,868.3
	270	28.6	9601.8
0.2	160	41.2	7822.1
	300	41.2	10,868.3
	270	28.6	9601.8
0.3	150	46.5	8650.5
	300	46.5	11,964.3
	270	18	10,046.5
0.4	150	46.5	8650.5
	300	46.5	11,964.3
	270	18	10,046.5
0.5	150	37	9094.1
	300	37	13,236.8
	240	37	10,969.8
0.6	150	37	9452.5
	300	37	13,567.7
	240	37	11,322.8
0.7	150	37	9810.9
	300	37	13,898.7
	240	37	11,675.8
0.8	150	37	10,169.3
	300	37	14,229.6
	240	37	12,028.8
0.9	150	37	10,527.8
	300	37	14,560.5
	240	37	12,381.8
1	150	37	10,886.2
	300	37	14,891.4
	240	37	12,734.9

the energy market without considering the TMSR market. So, the proposed method in Section 4.2 is valid for the power systems whose probability of calling reserve is less than 0.3 (the results in case 2 and 3 are identical). If we suppose or hope that the power systems have good enough reliability, the proposed method in Section 4.2 will be suitable for Gencos in different power systems with regard to their rules.

6. Conclusions

The history of electricity markets shows that these markets are not fully competitive. The finite number of power suppliers, the lack of enough transmission capacity, etc. are some reasons of this lack of achievement. So, each Genco or player in these markets should be able to choose a good bidding strategy in order to maximize its profit. Each Genco can obtain its desired profit by having a share in different markets such as the energy and ancillary service markets. Among the different ancillary service markets, the spinning reserve market has the most interdependence with the energy market, and they are dealt with simultaneously.

In this paper, a new approach is proposed for presenting the optimal bidding strategy of Gencos in the energy and TMSR markets. In this condition, Gencos should share their total capacity for two markets so as to maximize their profits. The difficulty of making good decisions about the behavior of Gencos in both the energy and TMSR markets is faced through a sequential solution based on the probability of calling reserve. Also, the proposed method is based on the behavior of participants while considering the ISO's objective function. The WSCC nine bus system is employed to illustrate the proposed method. The method can be implemented for determination of the market equilibrium points as well as a method for Gencos to present their bidding strategies. Market power monitoring is another application of the proposed method, and it can be easily extended to more complicated networks.

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